NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja
FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2025_2 EXAMINATIONS

Course Code: STT 311
Course Title: Probability Distribution II
Credit Unit: 3
Time Allowed: 3 HOURS
Total: 70 Marks
Instruction: Attempt question No. 1 and any other 3 questions

Question One

(a) Find the expectation of a discrete random variable Y whose probability function is given by $f(y) = \left(\frac{1}{3}\right)^y ; y = 1,2,3,\cdots$ **(5 marks)**

(b) Find the mean and variance of a continuous random variable Y with frequency density function $f(y) = 5e^{-5y}; y \geq 0$. **(8 marks)**

(c) Find the variance of the sum obtained in tossing a fair dice. **(5 marks)**

(d) The random variable Y can assume the values 1 and -1 with probability $\frac{1}{3}$ each. Find (i) The moment generating function (ii) The first four moments about the origin **(7 marks)**

Question Two

(a) State and prove the Chebychev Inequality of bound and limit theorem. **(4 marks)**
(b) A random variable Y has $\mu = 3$, $\sigma^2 = 1$ and an unknown probability distribution, using Chebychev inequality, find (i) $Prob(0 < Y < 6)$ (iii) $Prob(|Y - 3| \geq 2)$. **(4 marks)**
(c) State and prove the Chernoff Inequality of bound and limit theorem. **(3 marks)**
(d) Given the Chernoff bound of a normally distributed random variable Y with mean value $\mu = 5$ and variance $\sigma^2 = 3$, find the $Prob(Y \geq 8)$. **(4 marks)**

Question Three

(a) Differentiate between the weak law and strong law of large number. **(2 marks)**
(b) State and prove the central limit theorem. **(10 marks)**
(c) A fair coin is tossed 400 times. Use the central limit theorem to compute an approximation to the probability of getting at least 205 heads.**(3 marks)**

Question four

A random variable Y has density function given by $f(y) = \begin{cases} 4e^{-4y} & \\ 0 & otherwise \end{cases}$ $y \geq 0$, find

(a) The moment generating function and use it to generate the first four moments about the origin. **(7 marks)**

(b) Find the mean and variance of a Normal distribution of a random variable Y whose the frequency density function is given by $f(y) = \frac{1}{\sigma\sqrt{2\pi}} \exp -\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2, y \geq 0.$ **(8 marks)**

Question Five

(a) Find the mean and variance for a Binomial distribution whose probability of success is p and probability of failure is q. **(8 marks)**

(b) State and prove Demoivre's theorem **(7 marks)**

Question Six

(a) Find the mean and variance for a Poisson distribution Y with probability function $f(y) = \begin{cases} \frac{\lambda^y e^{-\lambda}}{y!}, & y = 0,1,2, \dots, \lambda > 0 \\ 0 & otherwise \end{cases}$. **(8 marks)**

(b) Given that Y is a random variable with probability function $f(y) = \frac{y}{3}, y = 0, 1, 2.$ Find the second and third moments about the mean. **(7 marks)**