NATIONAL OPEN UNIVERSITY OF NIGERIA
PLOT 91, CADASTRAL ZONE, NNAMDI AZIKIWE EXPRESSWAY, JABI - ABUJA
FACULTY OF SCIENCES
DEPARTMENT OF PHYSICS
2025_2 EXAMINATIONS

COURSE CODE: **PHY301**
COURSE TITLE: **CLASSICAL MECHANICS**
CREDIT UNIT: **3**
TIME ALLOWED: **(3 HRS)**
INSTRUCTION: ***Answer question 1 and any other three questions***

QUESTION ONE
(a) (i) Distinguish between a particle and a rigid body. **4 marks**
(ii) Given a system of N particles, what are the necessary and sufficient conditions for it to be classified as a rigid body? **4 marks**
(b) Define the following terms as applied in classical mechanics:
(i) number of degrees of freedom **2 marks**
(ii) constraint **2 marks**

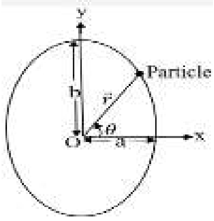


Figure 1: A particle sliding on an elliptical wire

(c) Consider a particle constrained to slide on an elliptical wire loop in the x-y plane as shown in Fig. 1, where a and b are the semi-minor and semi-major axes respectively and θ is the angle the radius vector $\vec{r}$ from the origin O to the particle makes with the positive x-axis.
(i) Derive the equation of the constraint on the motion of the particle. **7 marks**
(ii) State the correct classification of the constraint of the system. **2 marks**
(iii) Calculate the number of degrees of freedom of the motion of the particle. **4 marks**

QUESTION TWO
(a) (i) What is dot cancellation? **2 marks**
(ii) Show that $\frac{\partial \dot{\vec{r}}_i}{\partial \dot{q}_k} = \frac{\partial \vec{r}_i}{\partial q_k}$ **7 marks**
(iii) Proof the relation: $\frac{d}{dt}\left(\frac{\partial \vec{r}_i}{\partial q_k}\right) = \frac{\partial \dot{\vec{r}}_i}{\partial q_k}$ **6 marks**
(b) Consider a single particle moving in a circle on a plane surface.
(i) Write down the transformation equations that relate the Cartesian coordinates (x, y) to the plane polar coordinates (r, θ)
(ii) Write the transformation equations (b)(i) in terms of generalised coordinates using suitable symbols.

QUESTION THREE
(a) (i) Define virtual displacement, listing the conditions it must satisfy. **4 marks**
(ii) Given the transformation equations in terms of the generalised coordinates q_i as
$\vec{r}_i = \vec{r}_i(q_1, q_2, \dots, q_n, t),\ i = 1,2, \dots, 3N,$

where N is the number of particles and n the number of degrees of freedom, derive the the relationship between virtual displacement δr_i and infinitesimal changes in generalized coordinates δq_j, for $j = 1,2, \ldots, n$ and n is the degrees of freedom. **4 marks**

(b) (i) Explain why constraint forces do no virtual work when using generalized coordinates. **2 marks**

(ii) Derive the generalized forces starting from the expressions for virtual work and virtual displacement. Comment on your result. **5 marks**

QUESTION FOUR

(a) (i) In tabular form, briefly describe the differences between Newtonian Mechanics and Lagrangian Mechanics **4 marks**

(ii) What are generalized coordinates? Why are they useful in Lagrangian Mechanics? **3 marks**

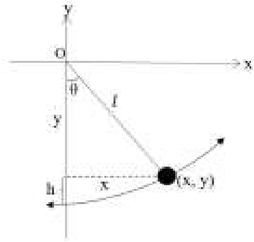


Figure 2: A simple pendulum

(b) A simple pendulum of mass m and length *l* swings under gravity (Fig. 2). Set up the Lagrangian and derive the equation of motion. **8 marks**

QUESTION FIVE

(a)(i) Explain the role of the Legendre transform in the formulation of Hamiltonian mechanics. **2 marks**

(ii) What is the physical meaning of the Hamiltonian for a conservative system? **2 marks**

(b) (i) Distinguish between Hamilton's and Lagrange's equations of motion. **3 marks**

(ii) Given the Lagrangian $L(q, \dot{q}) = a\dot{q}^2 + bq\dot{q}$, where a and b are constants. Find the Hamiltonian. **4 marks**

(iii) From the Hamiltonian in (a)(ii), obtain the equations of motion. **4 marks**

QUESTION SIX

(a)(i) What a central force? **1 mark**

(ii) Give two examples of a central force. **1 mark**

Figure 3: Two-particle central force system

(b) Consider the two-body central force system comprising two particles of masses m_a and m_b with displacements of $\vec{r}_a$ and $\vec{r}_a$ from the origin of coordinate system (Fig. 3). For this system,

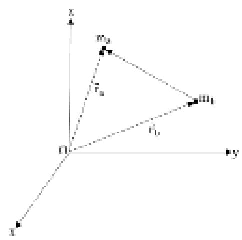
(i) write down the position vector of the centre of mass $\vec{R}$ and the reduced mass μ. **2 marks**

(ii) show that

$$\vec{r}_a = \vec{R} + \frac{m_b}{m_a + m_b}\vec{r}$$ **3 marks**

and

$$\vec{r}_b = \vec{R} - \frac{m_a}{m_a + m_b}\vec{r}$$ **3 marks**



(iii) derive the expression for the kinetic energy of the system **3 marks**

(iv) write down the Lagrangian for the system. **2 marks**