NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja
FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2025_2 EXAMINATIONS

Course Code: MTH411
Course Title: Measure Theory
Credit Unit: 3
Time Allowed: 3 Hours
Total: 70 Marks
Instruction: Answer Question One (1) and Any Other Three (3) Questions

Question 1

a. Define the following Outer Measure and Inner Measure
b. Explain the terms
   i. f is L-measurable set and
   ii. Length of a set A
c. State Beppo Levi theorem.
d. Suppose $f \in \mathscr{L}[a,b]$. Given any $\in > 0$, there exists a $\delta > 0$. Prove that if $m(E) < \delta$, then $\left|\int_E^{\cdot} f\right| < \in$.

Question 2

a. Define σ −algebra (sigma algebra). (2marks)
b. Prove that if A_1 and A_2 are measurable sets then $A_1 \cup A_2$ and $A_1 \cap A_2$ are measurable sets. (13marks)

Question 3

a. With a clearly stated definition distinguish between
   i. Upper sum and upper integral (4marks)
   ii. Lower sum and lower integral (4marks)
b. Show that if a closed set F is contained in an open interval Δ, then $m(F) = m(\Delta) - m(\Delta - F)$. (7marks)

Question 4

a. Explain the term simple function. (3marks)
b. Let $\{f_n\}$ be a sequence of non-negative functions in $\mathscr{L}[a,b]$ suppose $\inf\limits_{x \to \infty} f(x) = f(x)$ a.e in $[a,b]$ show that:
   i. If $f \in \mathscr{L}[a,b]$, then $\int_a^b f \leq \inf\limits_{x \to \infty} \int_a^b f_n$ (9marks)
   ii. If $f \notin \mathscr{L}[a,b]$, then $\lim\limits_{x \to \infty} inf \int_0^b f_n = +\infty$ (3marks)

Question 5

a. Explain bounded function and Lebesque integrable bounded function. (5 marks)
b. Show that the function $f(x) = \begin{cases} 1, & x \in [0,1], x \text{ is rational} \\ 0, & x \in [0,1], x \text{ is irrational} \end{cases}$ (10marks)
is Lebesque integrable but not Riemann integrable.

Question 6

a. State (i) Monotone Convergence theorem. (3 marks)
(ii) Dominated Convergence theorem. (3 marks)
b. Let (X, $\mathscr{M}$, fl) be a measure space, and let f and g be extended real-valued functions on X that are equal almost everywhere. If fl is complete and if f is measurable, explain that g is measurable. (9 marks)