NATIONAL OPEN UNIVERSITY OF NIGERIA

University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja

FACULTY OF SCIENCES

DEPARTMENT OF MATHEMATICS

2025_2 EXAMINATIONS

Course Code: MTH402
Course Title: Topology II
Credit Unit: 3
Time Allowed: 3 Hours
Total: 70 Marks
Instruction: Answer Question One (1) and Any Other Three (3) Questions

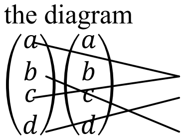
1a. Define a topology on X
b. Define convergent sequence.
c. (i) Define a dense Set and Everywhere Dense
(ii) Define a metrizable topological space
d. (i) Define Neighborhood of a point
(ii) Define Limit Point Compactness
e (i) Define Lebesque number of a Topological Space

2.Let (X,τ) and (Y,τ^*) be topological spaces.

a. When is a function $f:(X,\tau)\to(Y,\tau^*)$ said to be continuous?

b. Prove the result below:
Let $f:(X,\tau)\to(Y,\tau^*)$ and $g:(Y,\tau^*)\to(Z,\tau^{**})$ be continuous. Then, the composition function $g\circ f:(X,\tau)\to(Z,\tau^{**})$ is also continuous.

c. Consider the following topology defined on X = {a , b , c ,d }
$\tau=\{X,\emptyset,\{a\},\{b\},\{a,b\},\{b,c,d\}\}$. Let the function $f:(X,\tau)\to(X,\tau)$ be defined by the diagram



(i) Show that f is not continuous at c
(ii) Show that f is continuous at d.

3.Prove the result below
a. A topological space (X,τ) is connected if and only if it can not be expressed as the union of two non-empty sets that are separated in X.
b. Consider $\mathbb{R}$ with the left limit topology. Is the interval [0,1] connected?

4. Let X, Y be topological spaces with X compact and Y Hausdorff.
a. Prove that a continuous map $f:X\to Y$ must be closed.
b. Prove that a continuous bijection $f:X\to Y$ must be a homeomorphism.

5. Prove the result below
a. The real line $\mathbb{R}$ endowed with the standard topology is not compact.
b. Consider $\mathbb{R}$ with the left limit topology. Is the interval [0,1] compact?

6(a). Consider the topology $\tau=\{X,\emptyset,\{a\}\{a,c,d\},\{c,d\},\{b,c,d,e\}\}$ on $X=\{a,b,c,d,e\}$ and the subset $A=\{a,d,e\}$ of X. List the relative topology on A.

6.(b)Given that X and Y be two topological spaces. And let $f:X\to Y$ be a function. Suppose X is first countable. If for every sequence $\{x_n\}$ of X such that $x_n\to x$ in X as $n\to\infty$, one has that $f(x_n)\to f(x)$ in Y. Prove that$f$ is continuous.