NATIONAL OPEN UNIVERSITY OF NIGERIA

University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja

FACULTY OF SCIENCES

DEPARTMENT OF MATHEMATICS

2025_1 EXAMINATION

Course Code: MTH401
Course Title: GENERAL TOPOLOGY I
Credit Unit: 3
Time Allowed: 3 Hours
Total: 70 Marks
Instruction: Answer Question Number one and any Other Three(3) Questions

1a. Define the following:
i. Topological Space 2 marks
ii. Open cover of a Topological Space 2 marks
iii. Closed cover of a Topological Space 2 marks
iv. Homeomorphism of a function 2 marks
v. Compactness of a set 2 marks
b. Prove that a closed subset of a compact space is compact. 5 marks
c. Prove that homeomorphism preserves compactness. 8 marks
d. Is the space Q of rational numbers complete? 2marks

2a. Define the following:
i. A proper function. 2marks
ii. Bolzano-Weierstrass Theorem. 2marks
iii. What does it mean for a space to be metrizable? 2marks
b. Prove that a space is connected if and only if the only clopen sets are the empty set and the entire space. 4marks
c. Prove the Bolzano-Weierstrass Theorem. 5marks

3a. What is a neighborhood of a point in a topological space? 3marks
b. Define what it means for a topological space to be path-connected. 4marks
c. State and prove the Tychonoff Theorem. 8marks

4a. Define the following:
i. Heine-Borel theorem 2marks
ii. A continuous function between two topological spaces. 2marks
b. Prove Heine-Borel theorem. 7marks
c. Prove that the continuous image of a connected space is connected. 4marks

5ai. State Baire Category Theorem. 2marks
ii. Prove that the composition of two continuous functions is continuous. 4marks
b. Given a set X and a basis $\mathfrak{B}$ for a topology on X, prove that the topology generated by $\mathfrak{B}$ is unique. 3marks
c. State and prove Urysohn's Lemma. 6marks

6a. Every subsequence of a convergent sequence converges, and it converges to the same limit as does the mother sequence. (5 marks)

b. Let (E_1, d_1) and (E_2, d_2) be two metric spaces and let $E = E_1 \times E_2$ denote their cartesian product, where E is endowed with its own metric. Define Euclidean metric on $E_1 \times E_2$. (10 marks)