NATIONAL OPEN UNIVERSITY OF NIGERIA

University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja

FACULTY OF SCIENCES

DEPARTMENT OF MATHEMATICS

2025_2 EXAMINATIONS

Course Code: MTH401
Course Title: GENERAL TOPOLOGY I
Credit Unit: 3
Time Allowed: 3 Hours
Total: 70 Marks
Instruction: Answer Question One (1) and Any Other 3 Questions

1a. Define a topological space X and explain with an example. 7marks

b. Let $\{F_i\}_{i \in I}$ be a collection of closed sets in a topological space (X, τ). Prove that the intersection of any collection of closed sets is closed. 8marks

c. Define homeomorphism. 2marks

d. Given a bijective function $f: X \to Y$, between two topological spaces X and Y, prove that the homeomorphism between the spaces preserve compactness. 8marks

2a. Let $f: X \to Y$ be a continuous function. Prove that a continuous function f from a compact space to a Hausdorff space is closed and proper. 6marks

b. Define and prove the Heine-Borel theorem. 9marks

3a. Let $f: X \to Y$ be a continuous function. Prove that the continuous image of the connected space is connected. 7marks

b. Prove that every metric space is a Hausdorff space. 8marks

4a. Define compactness of a topological space, say X. 2marks

b. Prove that a closed subset of a compact space is compact. 5marks

c. Prove that the intersection of any collection of closed sets is closed. 8marks

5a. Define a topological property and give two examples of topological properties. 3marks

b. Prove that a space is connected if and only if the only clopen sets are the empty set and the entire space. 6marks

c. Define a basis of a topology and explain with an example. 6marks

6a. State the Pasting lemma on union of closed sets. 8marks

b. Let $f: R \to R$ be defined by 7marks

$$f(x) = x^2 + 1, \text{ if } x \le 0 \text{ and } f(x) = \frac{1}{2}(x+2), if\ x \ge 0.$$

Show that f is continuous on R.