



NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja
FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2025_2 EXAMINATIONS.

Course Code: MTH382

Course Title: Mathematical methods IV

Credit Unit: 3

Time Allowed: 3 Hours

Total: 70 Marks

Instruction: Answer *QUESTION NUMBER ONE* and any Other Three Questions

1. Solve the Laplace equation $U_{xx} + U_{yy} = 0$ in the rectangular $0 < x < a, 0 < y < b$ and which satisfied the boundary conditions.

$$U(x, 0) = 0, U(x, b) = 0, 0 < x < a$$

$$U(y, 0) = 0, U(a, y) = f(y), 0 < y < b$$

Where f is the given function on $0 < y < b$ **(25 marks)**

2. (a) Show that, the two definitions of gamma function are equivalent **(8 marks)**

(b) Show that $\Gamma(2z) = \frac{2^{2z-1}}{\sqrt{\pi}} \Gamma(z) \Gamma\left(z + \frac{1}{2}\right); (2z = 0, -1, 2, \dots)$ **(7 marks)**

3. (a) Prove that $j_0(z) = \frac{1}{2} \int_0^{2\pi} \cos(z \cos \theta) d\theta$ **(5 marks)**

(b) Prove that $\int_0^{2\pi} j_0(z \cos \theta) \cos \theta d\theta = \frac{\sin z}{z}$ **(10 marks)**

4. If $R(p) > 0, R(q) > 0$, show that $B(p, q) = \frac{r(p)r(q)}{r(p+q)}$ **(15 marks)**

5. (a) Briefly explain the periodic function **(7 marks)**
(b) Define differential equation with relevant examples **(8 marks)**

6. Use factorial notation to show that;

(a) $(a)_{2n} = 2^{2n} \left(\frac{a}{2}\right)_n \left(\frac{a+1}{2}\right)_n$ **(7 marks)**

(b) $(a)_n = \frac{\Gamma(a+n)}{\Gamma(a)}$ **(8 marks)**