Course Code: MTH382
Course Title: Mathematical methods IV
Credit Unit: 3
Time Allowed: 3 Hours
Total: 70 Marks
Instruction: *Answer Question NUMBER ONE and any Other Three Questions*

1. Solve the Laplace equation $U_{xx} + U_{yy} = 0$ in the rectangular $0 < x < a, 0 < y < b$ and which satisfied the boundary conditions.

   $U(x,0) = 0, U(x,b) = 0, 0 < x < a$

   $U(y,0) = 0, U(a,y) = f(y), 0 < y < b$

   Where f is the given function on $0 < y < b$ **(25 marks)**

2. (a) Show that, the two definitions of gamma function are equivalent **(8 marks)**

   (b) Show that $\Gamma(2z) = \dfrac{2^{2z-1}}{\sqrt{\pi}} \Gamma(z)\Gamma\left(z + \dfrac{1}{2}\right); (2z = 0, -1, 2, \dots)$ **(7 marks)**

3. (a) Prove that $j_0(z) = \dfrac{1}{2} \int\limits_0^{2\pi} \cos(z\cos\theta)d\theta$ **(5 marks)**

   (b) Prove that $\int\limits_0^{2\pi} j_0(z\cos\theta)\cos\theta d\theta = \dfrac{\sin z}{z}$ **(10 marks)**

4. If $R(p) > 0, R(q) > 0$, show that $B(p,q) = \dfrac{r(p)r(q)}{r(p+q)}$ **(15 marks)**

5. (a) Briefly explain the periodic function **(7 marks)**
   (b) Define differential equation with relevant examples **(8 marks)**

6. Use factorial notation to show that;

   (a) $(a)_{2n} = 2^{2n}\left(\dfrac{a}{2}\right)_n\left(\dfrac{a+1}{2}\right)_n$ **(7 marks)**

   (b) $(a)_n = \dfrac{\Gamma(a+n)}{\Gamma(a)}$ **(8 marks)**