NATIONAL OPEN UNIVERSITY OF NIGERIA

University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja

FACULTY OF SCIENCES

DEPARTMENT OF MATHEMATICS

2025_2 EXAMINATIONS

Course Code: **MTH381**

Course Title: **MATHEMATICAL METHODS III**

Credit Unit: **3**

Time Allowed: **3 Hours**

Total: **70 Marks**

Instruction: **Answer Question One (1) and Any Other 3 Questions**

QUESTION ONE

(a) If $F(u,v)$ and $G(u,v)$ are differentiable in a region, obtain the Jacobian determinant of the functions. (i) If $x=\frac{1}{2}(u^2-v^2)$ and $y=uv$, compute $\frac{\partial(F,G)}{\partial(u,v)}$ (ii) Obtain $J=\frac{\partial(u,v)}{\partial(x,y)}$, if $x=rcos\theta$ and $y=rsin\theta$. **8marks**

(b) State the Residue Theorem. **3marks**

(c) Determine the Laurentz series expansion of the function $f(z)=\frac{1}{(z-1)(z-2)}$. **9marks**

(d) Evaluate the integral using residue theorem $\int_c \frac{1+z}{z(2-z)}dz$, where c is the circle $|z|=1$ **5marks**

QUESTION TWO

(a) If $R(u)=(u-u^2)i+2^3j-3k$, find (i) $\int R(u)du$ (ii) $\int_1^2 R(u)du$. 7 **Marks**

(b) The acceleration of a particle at any time $t\geq 0$ is given by $a=\frac{dv}{dt}=12\cos 2ti-8\sin 2tj+16tk$. If the velocity v and displacement r are zero at $t=0$, find v and r at any time. **6 Marks**

(c) Define the integral express for the Stokes' theorem of **A.** **2 Marks**

QUESTION THREE

(a) What is the relation between vector field and functions. **3marks**

(b) Prove that $\nabla r^n = nr^{n-2}r$. **7marks**

(c) if $\vec{A}=(3x^2+6y)i-14yzj+20xz^2k$, evaluate the line integral

$\oint \vec{A}dr$ from $(0,0,0)\,to\,(1,1,1)$ along the curve C. **5marks**

QUESTION FOUR

(a) Define complex function. Let $w = f(z) = z^2 + 3z$, find u and v when $z = 1 + 3i$.

5 Marks

(b) Verify that $u = x^2 - y^2 - y$ is harmonic in the complex plane and find a conjugate harmonic function of v of u. **7 Marks**

(c) Without proof, state the Cauchy's integral theorem. Hence, evaluate $\int_0^{1+i} z^3 dz$.

3 Marks

QUESTION FIVE

(a) Given $f(x) = 9\cos 2x$ and $g(x) = 2\cos^2 x - 2\sin^2 x$, show that the function is linearly dependent. **3 Marks**

(b) Compute the integrals $\int_{-\pi/2}^{\pi/2} \int_0^{2cos\theta} r^2 dr d\theta$. **6 Marks**

(c) Evaluate the integrals $\int_0^1 \int_0^1 (x^2 + y^2) dx dy$. **6 Marks**

QUESTION SIX

(a) Define the dependence and independence of a function. **3marks**

(b) If $u(x) = \sin bx$ and $v(x) = \cos bx$ determine whether the functions $u(x)$ and $v(x)$ are linearly dependent or independent. **6marks**

(c) Using the transformation $x + y = u, y = uv$ show that $\iint [xy(1 - x - y)]^{\frac{1}{2}} dydx = \frac{2\pi}{105}$, integration being taken over the area of the triangle bounded by the lines $x = 0, y = 0$

6marks