NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja
FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2025_1 EXAMINATION

Course Code: MTH341
Course Title: Real Analysis
Credit Unit: 3
Time Allowed: 3 Hours
Total: 70 Marks
Instruction: Answer Question One (1) and Any Other 3 Questions

1a. Prove that a function f defined on an interval I, is derivable at a point c implies that it is continuous at the point c. **(8 marks)**
b. State Rolle's Theorem. **(4 marks)**
c. State the General Mean Value theorem. **(5 marks)**
d. Find the Maclaurin's series expansion of (i) Cos x (ii) e^x **(8 marks)**

2a. Determine the values for all a and b for which $\lim_{x\to 0} \frac{[x(a-\cos x)+b\sin x]}{x^3}$ exists and is equal to $\frac{1}{6}$. **(7 marks)**
b. When is a function said to be differentiable at a point? **(8 marks)**

3a. Evaluate $\lim_{x\to 4} \frac{1}{\log(x-3)} - \frac{1}{x-4}$ **(7 marks)**
b. State without prove, the inverse function theorem. **(8 marks)**

4a. Evaluate $\lim_{x\to 0^+} \frac{\log \tan 2x}{\log \tan x}$ **(7 marks)**
bi. Deduce Lagrange's mean value theorem from the generalized mean value theorem **(4 marks)**
bii. Deduce Cauchy's mean value theorem from the generalised mean value theorem $f(z) = \frac{1}{z}$ **(4 marks)**

5ai. State without prove Taylor's infinite series expansion of f(x). **(3 marks)**
aii) State without prove Maclaurin's infinite series expansion of f(x). **(4marks)**

5b. Verify Rolle's Theorem for the function f, defined by
i. $f(x) = x^3 - 6x^2 + 11x - 6$ for all $x \in [1,3]$ **(4 marks)**

ii. $f(x) = (x-a)^m(x-b)^n$ for all $x \in [a,b]$, $m,n \in \mathbb{N}$ **(4 marks)**

6a. Verify the hypothesis and conclusion of Lagrange's Mean Value theorem for the function $f(x) = \frac{1}{x}$ for all $x \in [1,4]$. **(7 marks)**
b. Given that f is a one-one continuous function on an open interval I and $J = f(I)$. If f is differentiable at $x_0 \in I$ and if $f'(x_0) \neq 0$, show that f^{-1} is differentiable at $y_0 = f'(x_0) \in J$ and $(f^{-1})'(y_0) = \frac{1}{f'(x_0)}$. **(8 marks)**