NATIONAL OPEN UNIVERSITY OF NIGERIA
Plot 91, Cadastral Zone, Nnamdi Azikwe Expressway, Jabi, Abuja

FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2025_2 EXAMINATIONS

Course Code: MTH 341 **Course Title: Real Analysis**
Credit Unit: 3 **Time Allowed: 3 Hours**
Instruction: Answer Question Number One and any Other Four Questions

1. (a) If $\lim_{x\to 2} \frac{4x^3+3}{(x^2-4)5} = \frac{k}{5}$. Find the value of k **(5 marks)**

   (b) State, without proof, the Maclaurin series **(5 marks)**

   (c) Define the following

   i. Minimum **(2 marks)**
   ii. Absolute minimum **(2 marks)**
   iii. Maximum **(2 marks)**
   iv. Absolute maximum **(2 marks)**

   (d) State, without proof, the extreme value theorem. **(4 marks)**

2. (a) Let f be a continuous function over the closed interval $[a, b]$ and differentiable over the open interval (a, b) such that $f(a) = f(b)$. Show that there exists at least one $c \in (a, b)$ such that $f'(c) = 0$. **(5 marks)**
   (b) Find the Maclaurin Series expansion of $f(x) = \ln(3 + 4x)$ **(7 marks)**
3. (a) Prove that $(4 - \sqrt{3})$ is an irrational number, given that $\sqrt{3}$ is an irrational number. **(5 Marks)**
   (b) Find C of Cauchy's Mean Value Theorem for the functions $\frac{1}{x}$ and $\frac{1}{x^2} \in [4,6]$. **(7 Marks)**

4. (a) Suppose that $f(x)$ is continuous and differentiable on $[6,15]$, if $f(6) = -2$ and $f'(x) \leq 10$, what is the largest possible value for $f(15)$? **(5 Marks)**
   (b) Let f be continuous over the dosed interval $[a, b]$ and differentiable over the open interval (a, b). Show that there exists at least one point $c \in (a, b)$ such that
   $f'(c) = \frac{f(b)-f(a)}{b-a}$ **(7 Marks)**
5. (a) Evaluate $\lim_{x\to\infty} \left(1 + \frac{1}{x}\right)^{4x}$ **(5 marks)**
   (b) Find the local maxima and minima values of $f(x) = x^4 - 3x^3 + 3x^2 - x$ **(7 marks)**

6. (a) Determine all the number(s) c which satisfy the conclusion of Mean Value Theorem for $h(z) = 4z^3 - 8z^2 + 7z - 2$ on $[2,5]$. **(7 marks)**

   (b) Verify Rolle's theorem for the function $f(x) = -x^2 + 5x - 5$ on a closed interval $[1,4]$. **(5 marks)**