NATIONAL OPEN UNIVERSITY OF NIGERIA

University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja

FACULTY OF SCIENCES

DEPARTMENT OF MATHEMATICS

2025_2 EXAMINATIONS

Course Code: MTH342
Course Title: Real Analysis
Credit Unit: 3
Time Allowed: 3 Hours
Total: 70 Marks
Instruction: Answer Question One (1) and Any Other Three (3) Questions

1. (a) State Rolle's Theorem. **(5 marks)**
   (b) Explain the algebraic interpretation of Rolle's Theorem. **(9 marks)**
   (c) Show that there is no real number K for which the equation $x^3 - 192x + K = 0$ has two distinct roots in [0, 5]. **(11 marks)**

2. If a function f is such that its derivative, f' is continuous and on [a, b] and derivable on]a, b[, then show that there exists a number c ∈]a, b[such that
   $f(b) = f(a) + (b - a)^2 f'(a) + (b - a)\frac{1}{2} f'(c)$ **(15 marks)**

3. (a) If two functions f and g are (i) continuous in [a, b], (ii) derivable in]a, b[and (iii) $f'(x) = g'(x)$ ∀ x]a, b[, then f – g is a constant function. **(6 marks)**
   (b) If a function f is (i) continuous on [a, b] (ii) derivable on]a, b[and
   (iii) $f'(x) > 0$ ∀ x ∈]a, b[, then f is strictly increasing on [a, b]. **(9 marks)**

4. Show that $\frac{x}{1+x} < \log(1 + x) < x$, ∀ x > 0. **(15 marks)**

5. (a) State Darboux Intermediate Value Theorem for Derivatives. **(10 marks)**
   (b) Verify Cauchy's mean value theorem for the functions f and g defined as
   $f(x) = x^2$, $g(x) = x^4$ ∀ x ∈ [3, 5]. **(5 marks)**

6. (a) When does a function f defined on an interval I said to have
   (i) Local or relative maxima **(2 marks)**
   (ii) Local or relative minima **(2 marks)**
   (iii) An extreme value (an extremum or a turning value) **(2 marks)**
   (b) Find the greatest and the least values of the function f
   defined by $f(x) = 6x^4 - 4x^3 - 12x^2 + 12x + 1$ in the interval [0, 3]. **(9 marks)**