NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja
FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2025_1 EXAMINATION

Course Code: MTH301
Course Title: Functional Analysis I
Credit Unit: 3
Time Allowed: 3 Hours
Total: 70 Marks
Instruction: Answer Question One (1) and Any Other Three (3) Questions

1. (a) Define a topology τ on X. **(4 marks)**
   (b) Give an example in each of discrete and indiscrete topology. **(5 marks)**
   (c) Let X be a complete metric space and $\{O_n\}$ is countable collection of dense open subsets of X. Show that $\bigcup O_n$ is not empty. **(11 marks)**
   (d) Let K ⊆ X be compact. Show that K is bounded. **(5 marks)**

2. (a) The collection Zd defined as $Zd = \{A \subseteq X: x \in A$ implies there exists $r > 0$ such that B(x,r) ⊆ A} is a topology on X, known as the topology induced by the given metric d. In a metric space (X, d) for each $x \in X$, r > 0, show that $B(x, r)$ is an open subset of (X, Zd). **(7 marks)**

   (b) Let K be a collection of nonempty closed subsets of a compact space T such that every finite sub collection of K has a nonempty intersection. Show that the intersection of all sets from K is non-empty. **(8 marks)**

3. (a) State Heine-Borel theorem. **(3 marks)**
   (b) Show that a continuous image of a compact space is compact. **(12 marks)**

4. (a) State axioms of addition of a real number system $(\Re, +, \cdot)$ **(5 marks)**
   (b) Prove that a subspace T of a topological space S is disconnected iff it is separated by some open subsets U, V of S. **(10 marks)**

5. Let $(X, d)$and (Y, d) be metric spaces and g is a mapping of X into Y. Let τ and τ_1 be the topologies determined by d and d_1 respectively. Show that $g(x, \tau) \to (y, \tau)$ is continuous if and only if $x_n \to x \Rightarrow g(x_n) \to g(x)$: that is if $x_1, x_2, \cdots x_n, \cdots$ is a sequence of points in (X, d)converging to x, then the sequence of points $g(x_1), g(x_2), \cdots g(x_n), \cdots$ in (Y, d)converges to x. **(15 marks)**

6. Prove that a set A is a closed set if and only if it contains all its limit points. **(15 marks)**