NATIONAL OPEN UNIVERSITY OF NIGERIA

University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja

FACULTY OF SCIENCES

DEPARTMENT OF MATHEMATICS

2025_2 EXAMINATIONS

Course Code: MTH301
Course Title: Functional Analysis I
Credit Unit: 3
Time Allowed: 3 Hours
Total: 70 Marks
Instruction: Answer Question One (1) and Any Other Three (3) Questions

1a. (i) Define a metric space. (5 marks)
(ii) $For\ all\ X, Y \epsilon R,\ d(x, y) = |X - Y|$. Verify the triangle inequality. (3 marks)
(iii) Convergence of metric space (3 marks)
(iv) Discrete metric space (3 marks)
b. (i) Show that a convergent sequence in a discrete metric space become constant after a finite number of terms (5 marks)
(ii) Explain why the closed interval [0,1] is a closed subset of R (3 marks)
(iii) Define the closure of a subset A of a metric space X. (3 marks)
2a. Show that $d(P_{1,}P_2) = |X_1 - X_2| + |Y_1 - Y_2|$ is satisfies the triangle inequality, where
$P_1 = (X_{1,}Y_1)\ ;\ P_2 = (X_2Y_2);\ X_1, X_2, Y_1, Y_2 \in R$ (5 marks)
b. State the following inequality:
(i) Cauchy-Schwarz Inequality. (3 marks)
(ii) Minkoski's Inequality (3 marks)
(iii) A bounded metric (3 marks)
3a. Define the following terms and give example of each
(i) an open sphere in a metric space. (3 marks)
(ii) a closed sphere in a metric space. (3 marks)
3b. (i) Let (X, d) be the discrete metric space, write open balls centred at $x \epsilon X$ with radius $\frac{1}{2}$ and $\frac{3}{2}$, then show that every subset of discrete metric space is closed. (3marks)
(ii) Show that sequence $\{X_n\} = \left\{\frac{3}{n}\right\}$ of real number converge to 3 (6marks)
4a. (i) Define topological spaces and give two (2) examples. (5marks)
b. (i) If a sequence $\{x_n\}$ in a metric space of X converge to x then show that every subsequence $\{x_n\}$ convergence to x.
(ii) If a sequence $\{x_n\}$ converge to x then show that $\{x_n\}$ converge to x
5a. If A is a subset of a metric space X, then prove that
$\bar{A} = \{x$: each open sphere centred at x intersects $A\}$ (9marks)
b. (i) Define the neighbourhood of a point in a metric space. (3 marks)
(ii) Show that the intersection of two neighbourhoods of a point is also its neighbourhood in a metric space (5marks)
6. (a) Define the distance between two sets and show with the help of an example that the distance between two non-empty disjoint sets may be zero. (10 marks)
(b) Define pseudo metric. (5 marks)