CIT 335: Computational Science & Numerical Methods
Credit: 3 units
TIME ALLOWED: 2½ Hours
INSTRUCTION: Answer Question 1 and any other THREE (3) Questions

Question 1

a. Discuss the notational device that can be used to treat continuous and discrete case simultaneously. (4 marks)
b. Enumerate the useful properties of inner product. (5 marks)
c. Write short note on the term orthogonal systems. (4 marks)
d. Give an elementary example of how computers perform backward error analysis? (3 marks)
e. Highlight the condition of THE problem:
$$f: \mathbb{R}^n \longrightarrow \mathbb{R}, \quad y = x_1 x_2 \dots x_n.$$ (6 marks)
f. Briefly describe the reason why a real number cannot in general be represented exactly in a computer? (3 marks)

Question 2

a. Discuss how a cancellation error can occur in the computation of algebraic identity:
$(a - b)^2 = a^2 - 2ab + b^2$; where a = 1.8, b = 1.7   (3 marks)
b. Derive a bound for the relative error in floating point representation.**(7 marks)**
c. Describe the basic features that differentiate the fixed-Point data structured numbers from the floating-point numbers. (5marks)

Question 3

a. Use a schematic diagram to illustrate a cancellation error phenomenon of arithmetic operations. (6 marks)
b. Briefly explain the three error types that affect the result of a numerical computation. **(3marks)**
c. Write short note on the following:
   a. Complex floating-point numbers (2 marks)
   b. How to avoid rounding errors in a computer system arithmetic operation (2 marks)
   c. Euclidean algorithm (2 marks)

Question 4

a. Considering an exact value of variable $\mathbf{a} = \sqrt{2}$, and its' approximate $\bar{\mathbf{a}} = 1.414$ to briefly describe the basic concepts of errors. **(7 marks)**
b. Derive the error propagation formula for Multiplication. **(5 marks)**
c. The unit roundoff in a binary floating-point system is $\mu = 0.5 \cdot 2 - t$, how many digit must we have in a decimal system with approximately the same unit roundoff? **(3 marks)**

Question 5

a. Summarize in a table, the types of approximation and their associated norms. (6 marks)
b. Describe the model of a machine arithmetic? (3 marks)
c. What is a complex floating-point number? **(2 marks)**
d. Discuss the term Symmetric rounding. (4 marks)

Question 6

a. Discuss extensively how researchers view the concept of real number, both in the field of mathematics and computer. **(7 marks)**
b. Calculate the rounding error (R_c) associated with the rounding up and chopping of following numbers:
   a. 1.23767
   b. 0.774501
   c. 6.3225
   d. 6.3235   **(8 marks)**