NATIONAL OPEN UNIVERSITY OF NIGERIA

University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja

FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2024_2 EXAMINATION

Course Code: MTH411 **Course Title:** Measure Theory and Integration
Credit Unit: 3 **Time Allowed:** 3 Hours
Instruction: Attempt Number One (1) and Any Other Four (4) Questions

1. (a) Define equivalent functions **(3 marks)**
   (b) Define the least upper bound of the measures of all closed sets. **(3 marks)**
   (c) Suppose a bounded set E is the union of a denumerable number of pairwise disjoint sets E_k. Show that $M^*(E) \leq \sum_k M^*(E_k)$ **(6 marks)**
   (d) Distinguish between a measurable function and a Borel function using four examples**.** **(10 marks)**

2. (a) Prove that if E is any countable set of real numbers, then $\mu^*(E) = 0$ **(6marks)**
   (b) Prove that if two subsets A and B of the real line are measurable, then so is $A \cap B$ **(6marks)**

3. (a) Explain vividly what is meant by the Lebesgue Outer Measure μ*(E) of a subset E of the real line **R** **(6marks)**
   (b) Prove that for any two subsets A and B of **R**, if A ⊂B, then μ*(A) ≤ μ* (B) **(6 marks)**

4. Explain the properties that hold almost everywhere in a measure space(X, $\mathscr{M}$, μ). **(12 marks)**

5. (a) State (i) Dominated Convergence theorem **(4 marks)**
   (ii) Monotone Convergence theorem. **(3 marks)**
   (b) Let (X, $\mathscr{M}$, μ) be a measure space, and let f and g be extended real-valued functions on X that are equal almost everywhere. If μ is complete and if f is measurable, explain what is meant by g is measurable. **(5 marks)**

6. (a) Let A and B be bounded sets such that $A \subseteq B$. Show that $M^*(A) \leq M^*(B)$ **(6marks)**
   (b) Let F = [a, b].Prove that F is a non – negative bounded closed set **(6 marks)**