NATIONAL OPEN UNIVERSITY OF NIGERIA

University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja

FACULTY OF SCIENCES

DEPARTMENT OF MATHEMATICS

2024_2 EXAMINATION

Course Code: MTH402 | **Course Title: GENERAL TOPOLOGY II**

Credit Unit: 3 | **Time Allowed: 3 Hours**

Total: 70 Marks

Instruction: Answer Question One (1) and Any Other 3 Questions

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1. (a) Explain each of the following topological terms:

   (i) $\tau_4 - space$ **(3marks)**

   (ii) Continuous function **(3marks)**

   (iii) Nowhere dense set **(2marks)**

   (iv) *Baire Space* **(2marks)**

   (b) Let X and Y be topological spaces, let $f: X \longrightarrow Y$. Then *show* that the following are equivalent:

   (i) f is continuous.
   (ii) For every subset A of X, one has $f(\bar{A}) \subset \overline{f(A)}$.
   (iii) For every closed set B of Y, the set $f^{-1}(B)$ is closed in X.
   (iv) For each $x \in X$ and each neighbourhood V of $f(x)$, there exists a neighbourhood U of x such that $f(U) \subset V$. **(12marks)**

2. (a) (i) Explain Equal Cardinality. **(2marks)**

   (ii) Prove that any second countable topological space X is separable. **(5marks)**

   (b) Let (X, τ) be a topological space. If X is second countable, then show that X is first countable.

   (9marks)

3. (a) Let X be a first countable topological space and A be a subset of X. Then prove with claim that if $x \in A$, there exists a sequence $\{x_n\}$ of A such that $x_n \longrightarrow x$ as $n \longrightarrow \infty$. **(8marks)**

   (b) If the topology on the range Y is given by a basis $\mathcal{B}$, then*show* that$f$ is continuous if and only if for any basis element $B \in \mathcal{B}$, the set $f^{-1}(B)$ is open in X. **(8marks)**

4. (a) Let X be a compact space and let $\{C_n, n \geq 1\}$ be a collection of nonempty closed sets such that $C_{n+1} \subset C_n$. Then show that$\bigcap_{n\geq 1} C_i = \emptyset$. **(6marks)**

   (b) If $f: X \longrightarrow Y$ is continuous, and if A is a subspace of X, then *show* that the restricted function $f|_A: A \longrightarrow Y$ is continuous. **(10marks)**

5. (a) Prove that the real line $\mathbb{R}$ endowed with the standard topology is not compact. **(8marks)**

(b) Let $h: \mathbb{R} \longrightarrow \mathbb{R}$ be defined by

$$h(x) = \begin{cases} \frac{x}{2}, & if \quad x \geq 0 \\ x, \quad if & x \leq 0 \end{cases}$$

then show that h is continuous. **(8marks)**