NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja

FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2024_2 EXAMINATION

Course Code: MTH382 **Course Title:** Mathematical Methods IV
Credit Unit: 3 **Time Allowed:** 3 Hours
Total: 70 Marks
Instruction: Answer Question One (1) and Any Other Three (3) Questions

QUESTION ONE

(a) i. What is an ordinary differential equation? **[2 Marks]**

ii. Given that $(x+1)\frac{dy}{dx} = y + x + x^2$ with $y = 2$ at $x = 1$. Solve the differential equation to show that$y = 4(3 - \ln 2)\ at\ x = 3$.. **[7 Marks]**

(b) Show that $y' = F[t, y], y(t_0) = y_0$, has a unique solution defined in the interval $(t_0 - r, t_0 + r)$, where $r < min\left(a, \frac{b}{M}, \frac{1}{K}\right)$. **[11 Marks]**

(c) Consider the initial value problem (IVP) $\frac{dy}{dt} = 2ty,\ y(0) = 1$, and apply the Method of Successive Approximations. **[5 Marks]**

QUESTION TWO

(a) Describe the functional relation of the gamma function. **[2Marks]**

(b) Show that the two definitions of gamma function are equivalent. **[9 Marks]**

(c) Show that$(\alpha)_n = \frac{\Gamma(\alpha+n)}{\Gamma(\alpha)}$. **[4Marks]**

QUESTION THREE

(a) Define the Bessel equation. **[2 Marks]**

(b) Determine the Legendre polynomial $P_3(x)$. **[5Marks]**

(c) Starting from the generating function of the Bessel function of the first kind $e^{\frac{1}{2}x\left(t-\frac{1}{t}\right)}=\sum_{n=-\infty}^{\infty}[t^n J_n(x)], n\in\mathbb{Z},$ show that $J_n(x) = (-1)^n J_{-n}(x)$.. **[8Marks]**

QUESTION FOUR

(a) When do we say a function is periodic? **[2Marks]**

(b) Solve the differential equation $\frac{\partial^2 u}{\partial x^2} = y(4x^2 - 1)$ given the boundary conditions that at $x = 0$, $\frac{\partial u}{\partial x} = \cos 2y$ and $u = \sin y$..
[9Marks]

(c) Find the general solution of the differential equation $X'' - 4X = 0$. **[4Marks]**

QUESTION FIVE

(a) What is a partial differential equation? **[2 Marks]**

(b) Define the boundary value equation for a wave equation? **[2 Marks]**

(c) Given a stretched string of length 50 cm which is set oscillating by displacing its mid-point a distance of 2 cm from its rest position and releasing it with zero velocity. Solve the wave equation assuming $c^2 = 1$and determine the resulting motion $u(x, t)$. **[11 Marks]**