NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja

FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2024_2 EXAMINATION

Course Code: MTH381 **Course Title:** Mathematical Methods III
Credit Unit: 3 **Time Allowed:** 3 Hours
Total: 70 Marks
Instruction: Answer Question One (1) and Any Other Three (3) Questions

QUESTION ONE

(a) (i) If $u = xyz,\ v = x^2 + y^2 + z^2,\ w = x + y + z$, find $J = \frac{\partial(u,v,w)}{\partial(x,y,z)}$

(ii) Given the functions: $u = x^2 + y^2, v = 2xy$, determine the Jacobian. **8marks**

(b) Define the Jacobian of a functions u and v hence find the Jacobian of $u = x + \frac{y^2}{x}$ and $v = \frac{y^2}{x}$. **5marks**

(c) Find the Laplace transform of $f''(t)$. **5marks**

(d) Let $\mathrm{H(s)} = \frac{1}{\left(\mathrm{S}^2 + w^2\right)^2}$ find $H(t)$. **7marks**

QUESTION TWO

(aDefine the dependence and independence of a function. **3marks**

(b) If $u(x) = \sin bx$ and $v(x) = \cos bx$ determine whether the functions $u(x)$ and $v(x)$ are linearly dependent or independent. **6marks**

(c)Evaluate the integral $\int_0^1\int_0^1 \left(x^2 + y^2\right) dydx$. **6marks**

QUESTION THREE

(a) Evaluate $\oint_c \vec{f}.\vec{dr}$ by Stoke's theorem where $\vec{F} = \left(x^2 + y^2\right)\hat{i} - 2xy\hat{j}$ and c is the boundary of the rectangle $x = \pm a,\ y = 0\ and\ y = b$. **5marks**

(b) Find the Fourier series expansion of periodic function of period 2π, defined by

$$f(x) = \begin{cases} x & \text{if } \frac{-\pi}{2} < x < \frac{\pi}{2} \\ \pi - x & \text{if } \frac{\pi}{2} < x < \frac{3\pi}{2} \end{cases}$$ **6marks**

(c) Evaluate $\int_0^2 \int_0^{\sqrt{2x-x^2}} \frac{x}{\sqrt{x^2+y^2}} dydx$ by changing to polar coordinate. **4marks**

QUESTION FOUR

(a)Find the volume cut off from the paraboloid $x^2 + \frac{y^2}{4} + z = 1 \, by \, the \, plane \, z = 0$. **6marks**

(b) Using Stoke's theorem or otherwise, evaluate $\int_c [(2x-y)dx - yz^2 dy - y^2 zdz]$ where c is the circle $x^2 + y^2 = 1$corresponding to the surface of sphere of what unit radius. **4marks**

(c) Using the divergence theorem show that $\iint_s \nabla(x^2 + y^2 + z^2)\vec{d}s = 6v$ where s is closed surface enclosing volume V. **5marks**

QUESTION FIVE

(a) State Green's and Stoke's theorems without proof. **4marks**

(b) The vector field $\overline{F} = x^2\hat{i} + z\hat{j} + yzx$is defined over the volume of the volume of the cuboid given by $0 \le x \le a, 0 \le y \le b, 0 \le z \le c,$ enclosing the surface. Evaluate the integral surface $\iint_s \vec{F}.ds$. **7marks**

(c) Evaluate $\iiint \frac{dxdydz}{x^2 + y^2 + z^2}$ throughout the volume of the sphere $x^2 + y^2 + z^2 = a^2$ **4marks**