NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja

FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2024_2 EXAMINATION

Course Code: MTH307
Course Title: Numerical Analysis II
Time Allowed: 3 Hours **Total:** 70 Marks
Instructions: ATTEMPT QUESTION ONE (1) AND ANY OTHER THREE (3) QUESTIONS

1. Find the least squares quadratic $x^2 + bx + c = 0$, which best fits the curve $y = \sqrt{x}$ over the interval $0 \leq x \leq 1$. **(22 marks)**

2. Fix a natural cubic spline to the data below and use it to estimate $f(55)$

x	25	36	49	64	81
$y = f(x)$	5	6	7	8	9

(16 marks)

3. Evaluate $\int_0^{\frac{\pi}{3}} \sin x \, dx$ with $h = \frac{\pi}{12}$, correct to 5 decimal places using the trapezoidal rule and hence, obtain the absolute error. **(16 marks)**

4. Approximate the solution to the partial differential equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \qquad 0 < x < 1, \qquad 0 < y < 1$$
$$u(0, y) = 0, \qquad u(1, y) = y, \qquad 0 \leq y \leq 1$$
$$u(x, 0) = 0, \qquad u(x, 1) = x, \qquad 0 \leq x \leq 1$$

Take the exact solution to be $u(x, y) = xy$. Use $h = k = 0.25$. **(16 marks)**

5. Solve the boundary value problem: $y'' = xy + 1; \;\; y(0) + y'(0) = 1, y(1) = 1$

using the second order finite difference method with $h = \frac{1}{2}$. **(16 marks)**