NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja

FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2024_2 EXAMINATION

Course Code: MTH301 **Course Title:** Functional Analysis
Credit Unit: 3 **Time Allowed:** 3 Hours
Total: 70 Marks
Instruction: Answer Question One (1) and Any Other Three (3) Questions

Q1(a)i) Define each of the following:
(i) Discrete topology **(3 marks)**
(ii) Indiscrete topology. **(2 marks)**
(b) i) Define a metric d on a set X. **(5 marks)**
ii) Suppose $\{x_n\}$ is a sequence in a metric space (X, d). Show that there exists at most on e point $x \in X$ such that $\{x_n\}$ converges to x. **(7 marks)**
(c) If (X, d) is a complete metric and Y is a closed subspace of X. Establish that (Y, d). **(5 marks)**

Q2 (a) Define neighborhood of x (**3 marks)** ii) subsequence of $\{x_n\}$ **(4marks)**
(b) Established that a subset A of X is open if and only if its complement A^c is closed in X. **(9 marks)**

Q3 (a) Define a sequence $\{f_n\}$ that converges uniformly to f. **(4 marks)**
(b) Let$\{f_n\}$be a sequence of continuous functions from a metric space (X, d) to a metric (Y, ρ). Suppose that $\{f_n\}$ converges to f from X to Y. Show that f is continuous. **(12 marks)**

Q4 (a) Define a compact metric space. When is a subspace said to be compact? **(4 marks)**
(b) State two (2) elementary properties of continuous functions between topological spaces. **(6 marks)**
(c) Suppose (X, d) is compact and Y is a closed subset of X. Establish that Y is compact. **(6 marks)**

Q5 (a) Define each of the following:
(i) a separation **(4 marks)** (ii) a connected subset Y of X **(4 marks)**
(b) Differentiate between connected and disconnected topological spaces. **(3 marks)**
(c) Show that if $\{A_i\}_{i\in I}$ is a family of connected subsets of X such that $\cap_{i\in I} A_I \neq \emptyset$, then $A = \cup_{i\in I} A_i$ is connected. **(5marks)**