NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja
FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS

2024_1 EXAMINATION

Course Code: MTH411
Credit Unit: 3
Total: 70 Marks
Time Allowed: 3 Hours
Instruction: Answer Question One (1) and Any Other 4 Questions

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1. (a) Define equivalent functions. **(4 marks)**
   (b) What is the outer measure of a bounded set E? **(4 marks)**
   (c) State Holder's inequality without proof. **(4 marks)**
   (d) Suppose a bounded set E is the union of a denumerable number of pair-wise disjoint sets E_k. Show that $M_*(E) \geq \sum_k M_*(E_k)$ **(10 marks)**

2. (a) What is a q – algebra? **(6 marks)**
   (b) Explain vividly what you understand by the Lebesgue Outer Measure μ*(E) of a subset E of the real line **R** **(6marks)**

3. (a) Without proof, state the Dominated Convergence theorem. **(6 marks)**
   (b) Let (X, $\mathscr{M}$, μ) be a measure space, and let f and g be extended real-valued functions on X that are equal almost everywhere. If μ is complete and if f is measurable, explain what is meant by g is measurable. **(6 marks)**

4. (a) Let $(X, \mathscr{M})$ be a measurable space, let A be a subset of X that belongs to $\mathscr{M}$, and let f and g be real - valued measurable functions on A. Show that $f \vee g$ and $f \wedge g$ are measurable. **(6 marks)**
   (b) Let A and B be bounded sets such that A ⊆ B. Show that $M^*(A) \leq M^*(B)$**(6 marks)**

5. (a) Define counting measure on a measurable space(X, M). **(5 marks)**
   (b) LetG_1 and G_2be open sets such that $G_1 \subseteq G_2$. Prove that m(G_1) ≤ m(G_2). **(7 marks)**

6. (a) When is a measurable space countably additive? And when is it finitely additive? **(6marks)**
   (b) Prove that the measure of a bounded closed set F is non – negative. **(6 marks)**

15 marks

Q6

a. Describe the major functions of the central processing unit? (4 marks)

b. What is a computer network and why is it important? (5 marks)

c. Explain the key difference among LAN, WAN, and MAN. (6 marks