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NATIONAL OPEN UNIVERSITY OF NIGERIA

University Village, Plot 91, Cadastral Zone, Nnamdi Azikiwe Express Way, Jabi-Abuja

FACULTY OF SCIENCES

DEPARTMENT OF MATHEMATICS

2024_1 EXAMINATION

Course Code: MTH401 | **Course Title: GENERAL TOPOLOGY 1**

Credit Unit: 3 | **Time Allowed: 3 Hours**

Total: 70 Marks

Instruction: Answer Question One (1) and Any Other 3 Questions

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1a. Let (E, d)be an arbitrary metric space and let $\{x_n\}$ be a Cauchy sequence in E. Then, $\{x_n\}$ is bounded. **(8 marks)**

b. Define the following:

i. Derived Set **(2 marks)**

ii. Minkowski Inequality **(2 marks)**

iii. Discrete Topology **(2 marks)**

c. Let (E, d) be a metric space and let $a \in E$ be fixed. Let $f: E \to R$ be defined by $f(x) = d(x, a)$, for all $x \in E$. Prove that f is uniformly continuous on E. **(5 marks)**

d. Prove that very compact subset of a metric space is closed and bounded. **(6 marks)**

2a. When is the pair (E, d) said to be a metric space? **(5 marks)**

b. Let (X, d_X) and (Y, d_Y) be metric spaces and $f: X \to Y$ be a map. Let $x_0 \in X$ be arbitrary but fixed. Then prove that the following are equivalent.

i. f is continuous at x_0.

ii. if $\{x_n\}$ is a convergent sequence in X such that $x_n \to x_0 \in X$, then $f(x_n) \to f(x_0) \in Y$ as $n \to \infty$. **(8 marks)**

c. Define the convergence of limit of a sequence. **(2 marks)**

3a. Let F be the subset of a metric space (E, d). Prove that F is closed in E if and only if its complement is open in E. **(5 marks)**

b. i. Define an interior point x on a metric (E, d) which has a subset O. **(2 marks)**

ii. Let (E, d) be a metric space and F be the subset of E. Define the Limit point of F on E.