



**NATIONAL OPEN UNIVERSITY OF NIGERIA**  
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja  
**FACULTY OF SCIENCES**  
**DEPARTMENT OF MATHEMATICS**  
**2024/1 EXAMINATION**

Course Code: MTH381  
Credit Unit: 3  
Total: 70 Marks  
Instruction: Answer Question One (1) and Any Other Three (3) Questions

Course Title: Mathematical Methods III  
Time Allowed: 3 Hours

**QUESTION ONE**

- (a) (i) If functions  $u, v, w$  of three independent variables  $x, y, z$ , define the Jacobian of the function. If  $u = xyz$ ,  $v = x^2 + y^2 + z^2$ ,  $w = x + y + z$ , find  $J = \frac{\partial(u,v,w)}{\partial(x,y,z)}$  **10 Marks**  
(ii) Given the functions  $u = x^2 + y^2$ ,  $v = 2xy$ , determine the Jacobian. **4 Marks**  
(b) Determine the Wronskian of set  $\{e^x, e^{-x}\}$ . **4 Marks**  
(c) If an entire function  $f(z)$  is bounded in absolute value for all  $z$ , then  $f(z)$  must be a constant. Thus prove the Liouville's theorem. **5 Marks**  
(d) Let  $f(z)$  be a function that is analytic inside a simple closed path  $C$  and on  $C$ , except for finitely many singular point  $z_1, z_2, \dots, z_k$  inside  $C$ , then  
$$\oint_C f(z) dz = 2\pi i \sum_{j=1}^k \text{Res } f(z)$$
  
Hence, prove the Residue theorem. **6 Marks**

**QUESTION TWO**

- (a) Given  $f(x) = 9 \cos 2x$  and  $g(x) = 2 \cos^2 x - 2 \sin^2 x$ , show that the function is linearly dependent. **3 Marks**  
(b) Compute the integrals  $\int_{-\pi/2}^{\pi/2} \int_0^{2 \cos \theta} r^2 dr d\theta$ . **6 Marks**  
(c) Evaluate the integrals  $\int_0^1 \int_0^1 (x^2 + y^2) dx dy$ . **6 Marks**

**QUESTION THREE**

- (a) If  $A = (3x^2 + 6y)i - 14yzj + 20xz^2k$ , evaluate  $\int_C A \cdot dr$  from  $(0,0,0)$  to  $(1,1,1)$  along the following paths  $C$ . (i)  $x = t, y = t^2, z = t^3$ . **4 Marks**  
(b) (i) the straight lines from  $(0,0,0)$  to  $(1,0,0)$ , then to  $(1,1,0)$ , and the to  $(1,1,1)$   
(ii) the straight line joining  $(0,0,0)$  and  $(1,1,1)$ . **6 Marks**  
(c) Find the total work done in moving a particle in a force field given by  $F = 3xyi - 5zj + 10xk$  along the curve  $x = t^2 + 1, y = 2t^2, z = t^3$  from  $t = 1$  to  $t = 2$ . **5 Marks**

**QUESTION FOUR**

- (a) Find the modulus and argument of the complex number  $\frac{(1+i)(2+i)}{(3-i)}$ . **6 Marks**  
(b) Find the  $n$ th root of  $\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$  if  $n = \frac{3}{5}$ . **6 Marks**