



NATIONAL OPEN UNIVERSITY OF NIGERIA
Plot 91, Cadastral Zone, Nnamdi Azikwe Expressway, Jabi, Abuja

FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2024 1 EXAMINATION

Course Code: MTH 341

Credit Unit: 3

Instruction: Answer Question Number One and any Other Four Questions

Course Title: Real Analysis

Time Allowed: 3 Hours

- (a) If $\lim_{x \rightarrow \infty} x \tan\left(\frac{1}{x}\right) = k$, find the value of k (6 marks)

(b) State Rolle's Theorem (4 marks)

(c) Find the least and the greatest value of the function $f(x) = 2x^3 - 3x^2 - 12x + 1$ on the interval $[-2, \frac{5}{2}]$. (8 marks)

(d) State the Cauchy mean value theorem (4 marks)
- (a) Evaluate $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^{4x}$ (5 marks)

(b) Verify Rolle's theorem for the function $f(x) = \sin x + \cos x - 1$ on a closed interval $[0, \frac{\pi}{2}]$. (7 marks)
- (a) Prove that $(5 - \sqrt{3})$ is an irrational number, given that $\sqrt{3}$ is an irrational number. (3 Marks)

(b) Find C of Cauchy's Mean Value Theorem for the functions $\frac{1}{x}$ and $\frac{1}{x^2} \in [1, 2]$. (9 Marks)
- (a) Find the Maclaurin series expansion of e^{3x} (8 Marks)

(b) Prove that the greatest integer function defined by $f(x) = [x], 0 < x < 3$ is not differentiable at $x = 1$ and $x = 2$ (4 Marks)
- (a) Evaluate $\lim_{x \rightarrow 0} \left(\cot x - \frac{1}{x}\right)$ (5 marks)

(b) Find the absolute maximum and absolute minimum values of $f(x) = \frac{x^2 - 4}{x^2 + 4}$ on the interval $[-4, 4]$. (7 marks)
- (a) Determine all the number(s) c which satisfy the conclusion of Mean Value Theorem for $h(z) = 4z^3 - 8z^2 + 7z - 2$ on $[2, 5]$. (6 marks)

(b) Prove that function f given by $f(x) = \sin\left(2x + \frac{\pi}{4}\right)$ is decreasing on $\left(\frac{3\pi}{8}, \frac{5\pi}{8}\right)$ (6 marks)