



NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja
FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2024 1 EXAMINATION-

Course Code: MTH312

Credit Unit: 3

Total: 70 Marks

Instruction: Answer Question One (1) and Any Other 3 Questions

Course Title: Abstract Algebra

Time Allowed: 3 Hours

Q1(a) (i) Establish that if f is an ideal of a ring R , then there exists a ring homomorphism

$f: R \rightarrow \frac{R}{f}$ whose Kernel is f . **(9 marks)**

(ii) Let $R = Z_8$, Show that $I = \{\bar{0}, \bar{4}\}$ is an ideal of R . **(5 marks)**

Hence construct the Cayley table for $+$ in $\frac{R}{I}$. **(5 marks)**

(b) Define each of the following:

(i) an even permutation $f \in S_n$. **(3 marks)**

(ii) an odd permutation $f \in S_n$. **(3 marks)**

Q2 (a) Define each of the following: (i) an isomorphism f . **(2 marks)**

(ii) the signature of $f \in S_n (n \geq 2)$. **(2 marks)**

(b) Suppose $f = (1,2,3) \in S_3$. Find Sign of f . **(4 marks)**

(c) If $H \trianglelefteq G, K \trianglelefteq G$. Establish that $HK \trianglelefteq G$. **(7 marks)**

Q3 (a) Define disjoint cycles and write an example of each of two cycles that is (i) disjoint (ii) not disjoint in S_4 . **(6 marks)**

(b) Prove that the set A_n of even permutations in S_n , forms a normal subgroup of S_n of order

$\frac{n!}{2}$. **(9 marks)**

Q4 (a) Given the Klein 4 – group K_4 . Show that both its subgroup $\langle a \rangle$ and $\langle b \rangle$ are normal. **(9 mark)**

(b) Prove that every subgroup of Z is normal in Z . **(6 marks)**

Q5 (a) Establish that the subgroup $\langle (12) \rangle$ of S_3 is not normal. **(6 marks)**

(b) Let H be a subgroup of a group G and

(i) $g^{-1}Hg \subseteq H \forall g \in G$

(ii) $g^{-1}Hg = H \forall g \in G$

Show that (i) and (ii). **(9marks)**

le				
Medium	Thick coaxial cable		Twisted pair	
Maximum length				
Topology				
Advantages		Low cost	Existing environment can use Hub and connect the stations	

6(a) State and define the 6 parameters used by Digital Signature Algorithm
[9 marks]

(b) Justify your recommendation of Elliptic Curve Cryptography (ECC) to Rivest, Shamir Adi (RSA) to a client
[6 marks]