1. Find a cubic approximation to e^x by using Chebyshev polynomials.   **(25 marks)**

2. Given a continuous function e^x for $x \in [-1,1]$, fit a linear polynomial $c_0 + c_1 x$ to e^x and determine its root mean square error.   **(15 marks)**

3. Integrate $y = \sqrt{x}$ within the limits 1.00 and 1.30 using Simpson's $\frac{1}{3}$-rule with 7 ordinates and working with 5 decimal places. Hence, estimate the error of this method.   **(15 marks)**

4. Obtain the solution to the partial differential equation at the first- and second-time levels only $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$ $0 \le x \le 1$, $t \ge 0$ where

$$u(0, t) = 1, u(x, 0) = 1 + x \text{ and } \left.\frac{\partial u(x,t)}{\partial t}\right|_{x=1} = 0$$

   Take $h = 0.2$ and $k = 0.02$. Use the classical explicit method.   **(15 marks)**

5. Solve the boundary value problem $(1 + x^2)y'' + 4xy' + 2y = 2$; $y(0) = 0, y(1) = \frac{1}{2}$ using
   (i)   First order finite difference method.   **(7 marks)**
   (ii)   Second order finite difference method.   **(8 marks)**