NATIONAL OPEN UNIVERSITY OF NIGERIA

University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja

FACULTY OF SCIENCES

DEPARTMENT OF MATHEMATICS

2024_1 EXAMINATION

Course Code: MTH 301
Course Title: Functional Analysis
Credit Unit: 3
Time Allowed: 3 Hours
Total: 70 Marks
Instruction: Answer Question One (1) and Any Other 3 Questions

Q1 (a) Define each of the following:

(i) a complete metric space (X, d). **(3marks)**

(ii) a complete subset. **(3marks)**

(b) Given the space R with the usual metric. Establish that it is complete. **(12 marks)**

(c) Show that if $\{A_i\}_{i\in I}$ is a family of connected subsets of X such that $\cap_{i\in I} A_i \neq \emptyset$, then $A = \cup_{i\in I} A_i$ is connected. **(7 marks)**

Q2 (a) (i) What is a topological space (X, τ)? **(4 marks)**

ii) Using the axiom of addition. Show that the system $(R, +, \cdot)$ is an ordered field. **(5marks)**

(b) Differentiate between open set and closed set. **(6marks)**

Q3 (a) Define each of the following:

(i) a separation **(3marks)**

(ii) a connected subset Y of X. **(3marks)**

(b) Show that if $\{A_i\}_{i\in I}$ is a family of connected subsets of X such that $\cap_{i\in I} A_i \neq \emptyset$, then $A = \cup_{i\in I} A_i$ is connected. **(5 marks)**

(c) Differentiate between connected and disconnected topological spaces. **(4marks)**

Q4 (a) Define each of the following:

i) a continuous function f at the point x_0. **(4 marks)**

ii) a continuous function f. **(3marks)**

(b) Show that if $f: X \to Y$ be a function from a metric space (X, d) to another metric (Y, ρ) and let $x_0 \in X$. Then f is continuous at x_0 if for every sequence $\{x_n\}$ such that $x_n \to x, f(x_n) \to f(x_0)$. **(8 marks)**