NATIONAL OPEN UNIVERSITY OF NIGERIA

Plot 91, Cadastral Zone, Nnamdi Azikwe Expressway. Jabi, Abuja.

FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2023_2 EXAMINATIONS

Course Code: **MTH401**
Course Title: **General Topology I**
Credit Unit: **3**
Time Allowed: **3 Hours**
Instruction: **Attempt Number One (1) and Any Other Four (4) Questions**

1. (a) Define the following terms: **(3 marks)**
   (i) condensing point
   (ii) accumulation point of F, where F is a subset of E of the metric space (E, d). **(3 marks)**

   (a) Let (E, d_E) be a metric space and let (Y, d_Y) be a subspace of X. Let A be subset of Y. Show that A is closed in Y if and only if there exists a set F which is closed in E such that $A = Y \cap F$. **(8 marks)**

   (b) Verify that d is a metric on the real line: $(R, |\cdot|)$, where R denotes the set of real numbers and $d : R \times R \to R$ be defined by $d(x, y) := |x - y|$ for all $x, y \in R$.. **(8 marks)**

2. Show that every compact subset of a metric space is closed and bounded. **(12 marks)**

3. Show that a point x of a metric space (E, d) is a cluster point of a sequence $\{x_n\}$ in E if and only if there exists a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ such that $x_{n_k} \to x$ as $k \to \infty$. **(12 marks)**

4. Prove that limits of a sequence are unique (i.e. if $\{x_n\}$ converges to both x and $x^{\prime}$ then $x = x^{\prime}$). **(12 marks)**

5. Prove that $\{x_n\}$ converges to x in E, if and only if $\{d(x_n, x)\}$ converges to 0 in R. **(12 marks)**

6. Prove that every convergent sequence in any metric space is Cauchy. **(12 marks)**