

FACULTY OF SCIENCES  
DEPARTMENT OF MATHEMATICS  
2023\_2 EXAMINATIONS\_

Course Code: MTH 382  
Course Title: Mathematical Methods IV  
Credit Unit: 3  
Time Allowed: 3 Hours  
Instruction: Answer Number One (1) and Any Other (4) Questions

1. (a) Use fractional notation to show that  $(\alpha)_{2n} = 2^{2n} \left(\frac{\alpha}{2}\right)_n \left(\frac{\alpha+1}{2}\right)_n$  (6 Marks)
- (b) We prove that  $\Gamma z = \lim_{m \rightarrow \infty} \int_0^x [e^{-t} - (1 - \frac{t}{n})^R] t^{z-1} dt$  (16 Marks)
2. Define the norm of differential equation on  $\mathbb{R}^n$  and show the  $f^1(x) = F[x, f(x)], f(x_0) = y_0$  has unique solution. (12 Marks)
3. (a) Why do we impose boundary conditions? (3 Marks)
- (b) Show that  $\Gamma(2z) = \frac{2^{2z-1}}{\sqrt{\pi}} \Gamma(z) \Gamma(z + \frac{1}{2}), 2z = 0, -1, -2, \dots$  (9 Marks)
4. Show that if  $R(p) > 0, R(q) > 0, B(p, q) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$  (12 Marks)
5. (a) State the telegraphic equation (3 Marks)
- (b) When do we say a function is periodic? (3 Marks)
- (c) Express the validity of Fourier's series in the interval  $-c \leq x \leq c$ . (3 Marks)
- (d) Explain Ordinary Differential Equation as a functional equation (3 Marks)
6. Prove that  $\Gamma(Z) = \lim_{n \rightarrow \infty} \int_0^x \left(1 - \frac{t}{n}\right)^n t^{z-1} dt$  (12 Marks)