Plot 91, Cadastral Zone, [illegible]

FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2023_2 EXAMINATIONS

Course Code: MTH 301
Course Title: Functional Analysis
Credit Unit: 3
Time Allowed: 3 Hours
Instruction: Answer Number One (1) and Any Other Four (4) Questions

1. (a) (i) Define a metric on a non-empty set X. **(4 marks)**
   (ii) Let (X , d) be a metric space. Define a ball B of radius r around a point x ∈ X. **(2 marks)**
   (b) State two examples of a metric space with metric defined. **(4 marks)**
   (c) Let (X, d) be a metric space. Show that a subset A of X is closed in (X, d) if and only if every convergent sequence of points in A converges to a point in A. (In order words, A is closed in (X, d) if and only if $a_n \to x$ where x ∈ X and a_n is a sequence of points in A ∀n implies that x ∈A). **(12 marks)**

2. (a) Define a separated set T ⊂S, where T is a subspace of a topological space S. **(2 marks)**
   (b) State the axioms of multiplication and order axiom of a real number system$(\Re, +, \cdot)$. **(10 marks)**

3. (a) Define an open ball in a metric. **(2 marks)**
   (b) Let (k, d) be a compact metric space. Show that every sequence in k has a convergent subsequence. **(10 marks)**

4. (a) Define a boundary point in a metric space **(2 marks)**
   (b) State true or false for each of the following statements:
   (i) Set $A \subset X$ is open if $B(x,\varepsilon) \subseteq A$.
   (ii) Finite intersection of open sets is open.
   (iii) Arbitrary union of open sets is not open.
   (iv) A set S is closed if its compliment is not open.
   (v) Arbitrary intersection of closed set is closed.
   (vi) Finite union of closed sets is closed.
   (vii) Interior of A is the union of all open subsets of A.
   (viii) A set S is closed if $\bar{S} = S$, where $\bar{S}$ is the closure of S
   (ix) Int A is the largest open subset of A.
   (x) A set, which contains an open set, has an empty interior **(10 marks)**