



**NATIONAL OPEN UNIVERSITY OF NIGERIA**  
**University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Expressway, Jabi, Abuja**  
**FACULTY OF SCIENCES**  
**DEPARTMENT OF MATHEMATICS**  
**2023\_1 POP EXAMINATION**

**Course Code:** MTH421  
**Course Title:** Ordinary Differential equations  
**Credit Unit:** 3  
**Time Allowed:** 3 Hours  
**Total:** 70 Marks  
**Instruction:** Answer Question Number One and any Other Three Questions

- (a) Solve the following initial value problems

  - $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 0, \quad y(0) = 4, \quad y'(0) = 1$  (9 marks)
  - $x^2 \frac{d^2y}{dx^2} - 6y = 0, \quad y(1) = 2, \quad y'(1) = 1$  (9 marks)

(b) Verify that equation  $3x(xy - 2)dx + (x^3 + 2y)dy = 0$  is exact and find the general solution (7 marks)
- Solve the non-homogenous linear differential equation

  - $\frac{d^2y}{dx^2} - 3 \frac{dy}{dx} + 2y = x^2$  (9 marks)
  - $\frac{dy}{dx} - y^2 = 1$  (6 marks)
- (a) Prove that  $x = 3e^{7t}, y = 2e^{7t}$  and  $x = e^{-t}, y = -2e^{-t}$  are solution of a system of differential equations (6 marks)

(b) Find the solution to the boundary value problem  $y'' + 4y = 0, y(0) = 1, y\left(\frac{\pi}{2}\right) = 0$  (9 marks)
- (a) Show that  $y = e^{-2x}$  is a solution to the equation  $y'' + y' - 2y = 0$  (6 marks)

(b) Solve the equation  $\frac{d^3y}{dx^3} - 2 \frac{d^2y}{dx^2} - \frac{dy}{dx} + 2y = 0$  (9 marks)
- (a) Verify that equation  $(4xy + 3y^2 - x)dx + x(x + 2y)dy$  is /not exact, then solve the equation (10 marks)



(b) let A be the matrix given by:  $A = \begin{pmatrix} 1 & 0 & 3 \\ 2 & 1 & 2 \\ 0 & 0 & 2 \end{pmatrix}$ .

Find

- (i) The eigenvalues **(2 ½ marks)**
- (ii) The generalized eigenspaces **(2 ½ marks)**

6. (a) Find the solution in explicit form for equation

$$\frac{dy}{dx} = \frac{3x^2+4x+2}{2(y-1)}, \quad y(0) = -1 \quad \textbf{(9 marks)}$$

(b) Find the eigenvalues and eigenfunctions of the Sturm-Liouville problem

$$\frac{dy^2}{dx^2} + \lambda y = 0, \quad 0 \leq x \leq L, \quad y(0) = 0, \quad y(L) = 0 \quad \textbf{(6 marks)}$$