NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi, Abuja

FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2023_1 POP EXAMINATION

Course Code: MTH402
Course Title: General Topology II
Credit Unit: 3
Time Allowed: 3 Hours
Total: 70 Marks
Instruction: Answer Question One (1) and Any Other 3 Questions

1. (a) Briefly describe the followings:
   (i) Discrete topology **(4 marks)**
   (ii) Indiscrete topology. **(4 marks)**

   (b) Verify the finite intersection $\bigcap_{k=1}^{n} U_k$ of the elements of τ are in τ. Given that X is a set and a topology on X be a collection τ of subsets of X. **(8 marks)**

   (c) Prove that τ equals the collection of all unions of elements of B. Given that X is a set and B is a basis for a topology τ on X. **(9 marks)**

2. (a) Let (d, X) be a metric topological space. Then prove that the collection of all

   r – ball $B_d(x, r)$, for $x \in X$ and $r > 0$ is a basis for a topology on X. **(7 marks)**

   (b) Attest that $x \in \bar{A}$ if and only if $\forall V \in N(x)$, $V \cap A = \emptyset$, where A is a subset of a topological space X. **(8 marks)**

3. (a) The topologies basis for τ and τ^0 on X are defined by $\boldsymbol{B}$ and $\boldsymbol{B^0}$ respectively on X. then prove that the followings are equivalent:

   (i) τ^0 is finer than τ; **(4 marks)**

   (ii) $x \in X$ and $B \in \boldsymbol{B} \subset x$ x,∃ $B^0 \in \boldsymbol{B^0}$ such that $x \in B^0 \subset \boldsymbol{B}$. **(5 marks)**

   (b) State whether each of the following functions is a homeomorphism or not:
   (i) f: R →R given by f(x) = 4x + 1. **(2 marks)**
   (ii) F: (-1, 1) →R given by $F(x) = \frac{x}{1-x^2}$. **(2 marks)**
   (iii) The identity map g: R_1 →R is bijective and continuous. **(2 marks)**

4. (a) Briefly describe the connection between T_1 – space, T_3 – space and regularity on a

topological space X. **(5 marks)**

(b) Prove that Hausdorff space is T_1 and the converse is not true. **(5 marks)**

(c) Prove that Q is a set of rational numbers and $\bar{Q} = R$ is a dense subset of R. **(5 marks)**

5. (a) Classify each of the followings into countable or uncountable set:

(i) Z **(1 mark)**

(ii) The image of a countable set under any map. **(1 mark)**

(iii) R. **(1 mark)**

(iv) The set $N^2 = \{(k, n) : k, n \in N\}$. **(1 mark)**

(v) The union of a countable family of countable sets. **(1 mark)**

(vi) Q. **(1 mark)**

(b) Let f is a homeomorphism, X is a compact and Y is Hausdorff. Then prove that $f : X \to Y$ is a continuous bijective function. **(9 marks)**

6. (a) Briefly explain the following terms:

(i) Covering and Open Cover **(3 marks)**

(ii) Compact Set **(3 marks)**

(iii) Subcover **(3 marks)**

(b) Prove that h is continuous if $h: R \to R$ is defined by

(i) $h(x) = \frac{x}{2}, if\ x \geq 0$ and **(3 marks)**

(ii) $h(x) = x, if\ x \leq 0$. **(3 marks)**