NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi, Abuja

FACULTY OF SCIENCES
DEPARTMATICS
2023_1 POP EXAMINATION

Course Code: MTH312
Course Title: Abstract Algebra
Credit Unit: 3
Time Allowed: 3 Hours
Total: 70 Marks
Instruction: Answer Question One (1) and Any Other 3 Questions

1a) Define the following terms:
(i) Normal subgroup of a group G **(3 Marks)**
(ii) Commutator of x and y, $x,y \in$ G **(3 Marks)**
(iii) a ring with identity **(3 Marks)**
(iv) an even permutation. **(3 Marks)**

b) Show that if $f: G_1 \longrightarrow G_2$ is a homomorphism, then
(i) Ker f is a normal subgroup of G_1. **(3 Marks)**
(ii) Im f is a subgroup of G_2 **(3 Marks)**

c)Let R be a ring and $a \in$ R. Show that the set aR = {ax: a ∈ R} is a subring of R. **(7 Marks)**

2a) If R_1 and R_2 are two rings and f: $R_1 \longrightarrow R_2$ is a ring homomorphism. Define the followings

(i) Im f **(2 Marks)**
(ii) Ker f **(2Marks)**
(iii) Ring isomorphism. **(2 Marks)**

bi) Given that f: $R_1 \longrightarrow R_2$ is a ring homomorphism, f is surjective and I is an ideal of R_1. Show that f(I) is an ideal of the ring R_2. **(4.5 Marks)**

bii) If I is an ideal of a ring R, show that there exists a ring homomorphism f: R ⟶ R/I whose kernel is I. **(4.5 Marks)**

3a) Define the following terms

(i) ideal of a ring **(3 marks)**

(ii) proper ideal of a ring **(3 marks)**

(iii) The ideal generated by $a_1, a_2, \cdots, a_n$, elements of a ring. **(3 marks)**

bi) Given that X is an infinite set and I is the class of all finite subsets of X. Show that I is an ideal of $\wp(X)$. **(3 Marks)**

bii) For any ring R and $a_1, a_2 \in R$. Show that $Ra_1 + Ra_2 = \{x_1a_1 + x_2a_2 \in R\}$ is an ideal of R.

(3 Marks)

4a) Show that $\mathrm{Aut}\mathbb{Z} \cong \mathbb{Z}_2$ **(7 marks)**

b) Show that any cyclic group is isomorphic to $(\mathbb{Z}, +)$ or $(\mathbb{Z}_n, +)$. **(8 marks)**

5a) Define the following terms

(i) Principal ideal **(3 Marks)**

(ii) Nilpotent **(3 Marks)**

(iii) Nil radical of R. **(3 Marks)**

5b) Given that $f, g \in S_n$, show that sign $(f \circ g) = (\text{sign } f)(\text{sign } g)$ **(6 Marks)**

6a). Explain the following terms

i.) when a permutation is called r-cyclic. **(2 marks)**

ii.) A transposition. (2 marks)

iii.) When two cycles are said to be disjoint. **(2 marks)**

b) Express each of the following permutations as products of disjoint cycles.

i. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 4 & 2 & 1 & 3 \end{pmatrix}$ ii. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 5 & 3 & 1 & 2 \end{pmatrix}$

iii. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 8 & 4 & 7 & 2 & 1 & 3 & 6 & 5 \end{pmatrix}$

(9 marks)