NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi, Abuja
FACULTY OF SCIENCES
Department of Mathematics
2023_1 POP EXAMINATION

Course Code: MTH307
Course Title: Numerical Analysis II
Credit Unit: 3
Time Allowed: 3 Hours
Total: 70 Marks
Instruction: Answer Question One (1) and Any Other 3 Questions

1. (a) Given the general second order Partial Differential Equation

$$L(u) = Au_{xx} + Bu_{xy} + Cu_{yy} - H(x, y, u, u_x, u_y) = 0$$

Classify the following equation into Parabolic, Elliptic and Hyperbolic.

i. $u_t = u_{xx}$

ii. $u_{tt} = u_{xx}$

iii. $u_{xx} + u_{yy} = 0$ **(5 marks)**

(b) When is Partial Differential Equation said to be Parabolic, Elliptic and Hyperbolic. **(5 marks)**

(c) Solve the Laplace equation $u_{xx} + u_{yy} = 0,$ subject to the boundary conditions $u(x,0) = 1, u(0, y) = 0, u(1, y) = 0, u(x,1) = 1;\ 0 \le x \le 1,\ 0 \le y \le 1.$ **(15 marks)**

2. (a) Express the function $x^3 + 2x^2 - x - 3$ in terms of Legendre polynomials. **(7 marks)**

(b) Find the fourth degree least square polynomials to $|x|$ over $[-1,\ 1]$ by means of Legendre polynomials. **(8 marks)**

3. (a) Show that $T_n(x)$ satisfies the differential equation $(1-x^2)y'' - xy' + n^2 y = 0$ **(7 marks)**

(b) Convert the first 5 terms of the Taylor series expansion for e^x into Chebyshev polynomials. **(8 marks)**

4. (a) Enumerate the classification of least square approximation method and explain each. **(7 marks)**

(b) Derive the least square formula for discrete data. **(8 marks)**

5. (a) Distinguish between Hermite polynomial and cubic spline. **(7 marks)**

(b) Find the cubic spline given the data below

x	0	2	4	6
y	1	9	41	41

where $M_0 = 0$ and $M_3 = -12$. **(8 marks)**

6. (a) Define Simpson's $\frac{1}{3}$ rule. **(7 marks)**

(b) Evaluate $\int_0^{\frac{\pi}{3}} \sin x dx$ with $h = \frac{\pi}{12}$, correct to 5 decimal places using Trapezoidal rule.

(8 marks)