NATIONAL OPEN UNIVERSITY OF NIGERIA
Plot 91, Cadastral Zone, Nnamdi Azikwe Expressway. Jabi, Abuja.
FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2022_2 Examination

Course Code: MTH412
Course Title: Functional Analysis II
Credit Unit: 3
Time Allowed: 3 Hours
Instruction: Answer Question One and any Other three Questions

1. (a) Briefly explain the following terms

(i) Inner product. **(4 marks)**

(ii) Hilbert Space. **(3 marks)**

(iii) Orthogonal elements **(3 marks)**

(iv) Orthonormal set **(3 marks)**

(b) (i) Show that an inner product space E becomes a normed linear space

when equipped with the norm $\|x\| =< x, x, >^{1/2}, \ \forall \ x \in E$ **(7 marks)**

(ii) Show the parallelogram law in inner product space **(5 marks)**

2. (a) Briefly Explain Fourier Coefficient of element in an inner product space. **(5 marks)**

(b) Show that $\sum_{k=1}^{n} |\langle x, v_k \rangle|^2 \leq \|x\|^2$ for any $x \in E$. **(10 marks)**

3. (a) Briefly explain the following terms:

(i) Riesz – Fischer theorem in orthogonal system **(4 marks)**

(ii) The projection Theorem **(4 marks)**

(b) Let $(E, < \Theta >)$ be inner product space and $x, y \epsilon E$, then prove the Cauchy – Schwartz

inequality in inner product space. **(7 marks)**

4. (a) Briefly explain the following terms

(i) Direct sum of subspaces of a vector space **(4 marks)**

(ii) Direct sum decomposition theorem **(3 marks)**

(iii) Orthogonal complement **(4 marks)**

(b) Let U and V be arbitrary subspaces of a Hilbert space H. Prove that

(i) $U^{\perp}$ is a closed subspace of H. **(2 marks)**

(ii) $V^{\perp\perp\perp} = U^{\perp}$ **(2 marks)**

5. (a) What are the following operators in Hilbert spaces

(i) Adjoint operator **(3 marks)**

(ii) Unitary operator in Hilbert spaces **(2 marks)**

(iii) Self Adjoint **(2 marks)**

(b) Let $\{u_1, u_2, \ldots, u_n\}$ be an orthogonal basis of an inner product E. Prove that for any $v \in E$

$$v = \frac{\langle v,u_1\rangle}{\langle u_1,u_1\rangle}u_1 + \frac{\langle v,u_2\rangle}{\langle u_2,u_2\rangle}u_2 + . \;\; . \;\; . + \frac{\langle v,u_n\rangle}{\langle u_n,u_n\rangle}u_n$$ **(8 marks)**

6. (a) State the Bessel's inequality in an inner product space. **(2 marks)**

(b) Show that u^* is unique such that $u^* \epsilon U$ is a unique minimizing vector if and only if $(x - u^*)$ ⊥U

(10 marks)