NATIONAL OPEN UNIVERSITY OF NIGERIA
Plot 91, Cadastral Zone, Nnamdi Azikwe Expressway. Jabi, Abuja.
FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2022_2 Examination

Course Code: **MTH411**
Course Title: **Measure Theory and Integration**
Credit Unit: **3**
Time Allowed: **3 HOURS**
Instruction: **ATTEMPT NUMBER ONE (1) AND ANY OTHER THREE (3) QUESTIONS**

1. (a) State Fatou's Lemma **(3 marks)**
   (b) Let F = [a, b], S = [a, b] and $C_sF = \emptyset$. Evaluate $m(F)$ **(4 marks)**
   (c) Prove that F is a non – negative bounded closed set **(8 marks)**

   (d) Let G be a bounded open set such that G = $\cup_k G_k$. Prove that
   m(G) ≤ $\sum_k m\,(G_k)$. **(10 marks)**

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2. (a) State Holder's inequality. **(4 marks)**

   (b) Let (X, M) be a measurable space. Evaluate the point mass concentrated at x, such that x ∈ X and fl ∈ M? **(4 marks)**
   (c) Let (X, fl) have finite measure. Prove that $L^p \subseteq L^R$,where $1 \leq r < p < \infty$.
   (7 marks)

3. (a) Briefly Explain a q – algebra. **(7 marks)**
   (b) Prove that $m(G) \geq \sum_{k=1}^{n} M(I_k)$ over disjointed open intervals G. **(8 marks)**

4. (a) What are the four conditions f must be satisfied on the measurable function
   $f: A \rightarrow [-\infty, +\infty]$? **(8 marks)**

   (b) Let f and g be measurable functions on A. Prove that $f \vee g$ and $f \wedge g$ are measurable. **(7 marks)**

5. (a) State the following theorems

    (i) Monotone Convergence theorem. **(4 marks)**

    (ii) Dominated Convergence theorem. **(5 marks)**

   (b) State the four properties of the collection Ω of subsets of X called algebra. **(6 marks)**

6. Let (X, M) be a measurable space and let it be a finitely additive measure on (X, M). Prove that it is a measure if either

   (i) $\lim_k$ fl(A_k) = fl($\bigcup_k A_k$) holds for each increasing sequence {A_k } of sets that belong to M.

   (ii) $\lim_k$ fl(A_k) = 0 holds for each decreasing sequence {A_k} of sets that belong to M and satisfy $\bigcap_k A_k = \emptyset$. **(15 marks)**