NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja
FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2022_2 Examinations

Course Code: MTH402
Course Title: General Topology II
Credit Unit: 3
Time Allowed: 3 Hours
Total: 70 Marks
Instruction: Answer Question One (1) and Any Other 3 Questions

SET TWO

1. (a) Briefly Explain a basis for a topology on X. **(7 marks)**
(b) Let X be a topological space and C is open of subsets of a set X. Prove that C is a basis for a topology on X?
(4 marks)
(c) Verify the finite intersection $\bigcup_{i=1}^{n} U_i$ of the elements of τ are in τ. Given that X is a set and a topology on X be a collection τ of subsets of X. **(6 marks)**
(d) Prove that if $\boldsymbol{A}$ and $\boldsymbol{B}$ are basis for the topology on X and Y respectively, then the collection $C = \{A \times B : A \in \boldsymbol{A}$ and $B \in \boldsymbol{B}\}$ is a basis for the topology on $X \times Y$.
(8 marks)

2. (a) Briefly explain a metric on a set X. **(4 marks)**
(b) Let X and Y be two topological spaces, such that $\boldsymbol{B}: = \{U \times V: U$ is open in X and V is open in $Y\}$. Then, prove that **B** is basis for topology on $X \times Y$. **(4 marks)**
(c) Prove that $S = \{\pi\_1\text{^}(-1)\ (U): U\ is\ open\ in\ X\} \cup \{\pi_2^{-1}(V): V\ is\ open\ in\ Y\}$ is a sub-basis for the product on $X \times Y$. **(7 marks)**

3. (a) Given A is a subset and A^0 is the set of limit points of A and $\bar{A}$. Prove that $\bar{A} = A \cup A^0$. **(6 marks)**
(b) Prove that if X is a Hausdorff space, $\forall x \in X$, the singleton set $\{x\}$ is closed.
(9 marks)

4. (a) Briefly explain the following terms:
   (i) Topology generated by a basis **(4 marks)**
   (ii) Basis for a topology on a set X. **(4 marks)**
   (b) If the topology on the range Y is given by a basis **B**, show that f is continuous if and only if any basis element B∈ ***B***, the set f^{-1} (B) is open in X. (6 marks)

5. (a) Classify each of the following into Hausdorff space or not:
   (i) Every metric topology. **(1 mark)**
   (ii) Every discrete space. **(1 mark)**
   (iii) The real line R with the finite complement topology. **(1 mark)**
   (iv) R with the finite complement topology. **(1 mark)**

   (b) Let $(C_i)_{i\in I}$ be a collection of connected spaces on X and let p be a point of $\boldsymbol{T}_{i\in I}C_i$. Prove that the Union of $(C_i)_{i\in I}$ is one point in common is connected. **(11 marks)**

6. (a) Prove that $\mathcal{R}$ with the standard topology is not compact. **(6 marks)**
   (b) Let W be an open set in Z and $f : X \to Y$ and $g : Y \to Z$ are continuous. Prove that the map $g \circ f : X \to Z$ is continuous. **(6 marks)**

   (c) State the tube lemma. **(3 marks)**