NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi, Abuja

FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2022_2 Examinations

Course Code: MTH341
Course Title: Real Analysis
Credit Unit: 3
Time Allowed: 3 Hours
Total: 70 Marks
Instruction: Answer Question One (1) and Any Other 3 Questions

1a. If $f(x) = x^2$, defined on the interval $[a, b]$Show that $f'(c)$ exists if and only if $Lf'(c)$, $Rf'(c)$ exists and $Lf'(c) = Rf'(c)$. **(6 marks)**

b. State without prove the Taylor's Theorem with Schlomilch and Roche form of remainder. **(5 marks)**

c. Show that using Maclaurin's theorem, $\text{Cos } x \geq 1 - \frac{x^2}{2}$, for all $x \in \mathbb{R}$ **(8 marks)**

d. When is a function f said to be an increasing function in an interval? **(6 marks)**

2a. State without prove the Lagranges Mean Value Theorem. **(7 marks)**

b. Determine the values of a and b for which $\lim\limits_{x \to 0} \frac{[x(a - \text{Cos } x) + b\text{Sin } x]}{x^3}$ exists and is equal to $\frac{1}{6}$.

(8 marks)

3a. State without prove the Maclaurin's Theorem with Lagranges form of remainder

(5 marks)

b. Evaluate $\lim\limits_{x \to 0^+} \frac{\log \tan 2x}{\log \tan x}$ **(10 marks)**

4a. Deduce the two special form of remainders of Taylor's Theorem with Schlomilch and Roche form of remainder. **(9 marks)**

b. **Find the** $\lim_{x\to 4}\left\{\frac{1}{\log(x-3)} - \frac{1}{x-4}\right\}$. **(6 marks)**

5a. Verify Rolle's theorem for the function f defined by $f(x) = x^3$-$6x^2 + 11x - 6$ for all $x \in [1,3]$.**(8 marks)**

b. Show that if f is differentiable in $]a,b[$ and $f'(x) \neq 0$, for all $x \in]a,b[$, then $f'(x)$ retains the same sign, positive or negative, for all $x \in [a,b]$.**(7 marks)**

6a. Let a function f: $[0,5] \longrightarrow \mathbb{R}$ defined by

$$f(x) = \begin{cases} 2x + 1, \text{when } 0 \leq 3 \\ x^2 - 2, \text{when } 3 \leq x \leq 5 \end{cases}$$

Is the function f derivable at $x = 3$?**(5 marks)**

b. Verify the hypothesis and conclusion of Lagrange's mean value theorem for the functions:

(i). $f(x) = \frac{1}{x}$ for all $x \in [1,4]$ **(6 marks)**

(ii). $f(x) = \log x$ for all $x \in [1, 1 + \frac{1}{e}]$ all $x \in [2,4]$. **(4 marks)**