NATIONAL OPEN UNIVERSITY OF NIGERIA
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FACULTY OF SCIENCES
January\February Examination 2018

Course Code:	**STT311**
Course Title:	**Probability Distribution II**
Credit Unit:	**3**
Time Allowed:	**3 HOURS**
Instruction:	**ATTEMPT NUMBER ONE (1) AND ANY OTHER FOUR (4) QUESTIONS**

1. (a) What is probability space? **(2 Marks)**

   (b) A Statistics student who does not believe in hard work wants to make money and achieve success in life by playing lottery. This addicted gambler spins a wheel of fortune for several years and discovered that the experiment can be modelled in terms of a random variable X with density function

$$f(x)=\begin{cases} x, 0<x<1 \\ 2-kx, 1\le x<2 \\ 0, elsewhere \end{cases}$$

   i. Find the value of k that makes it a valid density function. **(4 Marks)**
   ii. What is the mean of the random variable X? **(4 Marks)**

   (c) In a statistical experiment, n people throw their hats in a box then each picks up one hat at random. What is the expected value of X , the number of people that get back their own hat? **(6 Marks)**

   (d) If there are 2 people that threw their hats in that box, find the approximate value for the expected number of people that get their hats back. **(6 Marks)**

2. (a) State any two axioms of probability. **(2 Marks)**

   (b) Given that the a continuous random variable X has the following probability density function: $f(x)=\begin{cases}\frac{x^2}{3}, -1<x<2\\ 0, otherwise\end{cases}$

Find $F(x)$ and use it to evaluate $P(0<x\leq 1)$ **(5 Marks)**

(c) The length of life measure in hours of a certain rate type of insect is a random variable X with probability density function:

$$f(x)=\begin{cases}\frac{3}{4}(2x-x^2), 0<x<2\\ 0, elsewhere\end{cases}$$

If the amount of food measured in milligrams consumed in a lifetime by such an inset is defined by the function $g(x)=x^2$, where x is the length of life measured in hours, find the expected amount of food that will be consumed by an insect of this type. **(5 Marks)**

3. (a) The joint density for the random variables X and Y where is

$$f(x,y)=\begin{cases}10xy^2, 0<x<y<1\\ 0, elsewhere\end{cases}$$

Find the:

(i) marginal densities $g(x)$ and $h(y)$ **(3 Marks)**

(ii) conditional density $f(y/x)$ **(3 Marks)**

(b) (i) In a gambling game, a man paid ₦5 if he gets all heads or tails when three coins are tossed and he will pay ₦3 if either one or two heads show. **(2 Marks)**

(ii) What is his expected gain? **(1 Mark)**

(iii) Do you think the game is fair according to your expected gain? Give reasons? **(3 Marks)**

4. (a) Define the moment generating function of a random variable X. **(2 Marks)**

(b) Does moment generating function always exist? Give reason. **(4 Marks)**

(c)The P.M.F of a distribution is given as $p(X=x)=\frac{m!}{x!(m-x)!}p^x q^{m-x}, \forall x=0,1,2,3,\ldots,m.$

Find the moment generating function of the distribution and use it to obtain the mean.

(6 Marks)

5. (a) (i) Define a random variable and state the types of random variables that we have. **(2 Marks)**

(ii) Given that X has the moment generating function $M_{x(t)} = \frac{1}{6}e^{-2t} + \frac{1}{3}e^{-t} + \frac{1}{4}e^{t} + \frac{1}{4}e^{2t}$.

Find $P(|X| \leq 1)$. **(3 Marks)**

(b) (i) Define characteristic function of a random variable Y.

(ii)Mention one advantage of characteristic function over moment generating function.

(iii) If X is a random variable with an exponential ∂f with parameter λ.
Find the characteristic function. **(7 Marks)**

6. (a) (i) State Chebyshev's inequality theorem **(2 Marks)**

(ii) Proof the Chebyshev's inequality **(3 Marks)**

(b) (i) Define the term conditional probability. **(2 Marks)**

(ii) The joint density function of X and Y is given as

$$f(x) = \begin{cases} 2e^{-x}e^{-2y}, 0 < x < \infty, 0 < y < \infty, 1 \leq x < 2 \\ 0, otherwise \end{cases}$$

Find the $p(X < a)$ **(5 Marks)**