



**NATIONAL OPEN UNIVERSITY OF NIGERIA**  
**Plot 91, Cadastral Zone, Nnamdi Azikiwe Expressway, Jabi, Abuja.**

**FACULTY OF SCIENCES**  
**April Examination 2019**

**Course Code:** STT311  
**Course Title:** Probability Distribution II  
**Credit Unit:** 3  
**Time Allowed:** 3 HOURS  
**Total:** 70 Marks  
**Instruction:** ATTEMPT QUESTION NUMBER ONE AND ANY OTHER FOUR (4) QUESTIONS

1. (a) Give a brief definition of the following terms:

- |                              |                         |           |
|------------------------------|-------------------------|-----------|
| i. Random experiment         | iv. Probability Measure |           |
| ii. Sample Space             | v. Random Variable      |           |
| iii. Event of a Sample Space |                         | (5 marks) |

(b) Let X be a random variable with probability density function.

$$f(x) = \begin{cases} c(1 - x^2) - 1, & -1 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$

What is the value of c?

(7 marks)

(c) Given the joint probability distribution

$$f(x, y, z) = \frac{xyz}{108} \quad \text{for } x = 1, 2, 3; \quad y = 1, 2, 3; \quad z = 1, 2$$

find

- the joint marginal distribution of X and Y;
- the joint marginal distribution of X and Z;
- the marginal distribution of X;
- the conditional distribution of Z given X = 1 and Y = 2;
- the joint conditional distribution of Y and Z given X = 3

(10 marks)

2. (a) The probability density function of X, the lifetime of a certain type of electronic device

(measured in hours), is given as  $f(x) = \begin{cases} \frac{a}{x^2} & x > 10 \\ 0 & x \leq 10 \end{cases}$

- Find the value a
  - Find P(X > 20)
- (b) Compute E(X) and Var(X), if X has a density function given by

6 Marks

$$f(x) = \begin{cases} \frac{1}{4}xe^{-\frac{x}{2}} & x > 0 \\ 0 & \text{elsewhere} \end{cases}$$

(6 marks)

3. (a) The density function of  $X$  is given by  $f(x) = \begin{cases} a + bx^2 & 0 \leq x \leq 1 \\ 0 & \text{elsewhere} \end{cases}$  and  $E(X) = 0.6$ ,

i. Find  $a$  and  $b$ .

ii. Find  $E(X^2)$

(4 marks)

- (b) The lifetime in hours of an electronic tube is a random variable having a probability density function given by  $f(x) = \begin{cases} xe^{-x} & x \geq 0 \\ 0 & \text{elsewhere} \end{cases}$ .

Compute the expected value lifetime of such a tube

(2 marks)

- (c) Derive the moment generating function for a discrete random variable  $X$  with the following density function:

$$f(x) = \frac{e^{-\lambda} \lambda^x}{x!}, \quad x = 0, 1, 2, \dots$$

(6 marks)

4. (a) Given that events  $A$  and  $B$  are independent and that  $P(A|B) = 0.2$  and  $P(B|A) = 0.5$ . Compute the probability  $P(A \cup B)$ .

(2 marks)

- (b) i. If two events,  $A$  and  $B$ , are such that  $P(A) = 0.5$ ,  $P(B) = 0.3$ , and  $P(A \cap B) = 0.1$  Find  $P(B|A)$   
ii. You are given  $P(A \cup B) = 0.65$  and  $P(A \cup B') = 0.85$ . Determine  $P(A)$

(5 marks)

- (c) Given that  $X_1$  and  $X_2$  are two events such that  $P(X_1) = 0.45$ ,  $P(X_1 \cup X_2) = 0.68$ . Find  $P(X_2)$ , when

- i.  $X_1$  and  $X_2$  are mutually exclusive  
ii.  $X_1$  and  $X_2$  are independent.

(5 marks)

5. (a) Determine the value of  $k$  for which the function given by

$$f(x, y) = kxy \quad \text{for } x = 1, 2, 3, 4; \quad y = 1, 2, 3, 4$$

can serve as a joint probability distribution.

(6 marks)

- (b) If the joint probability density of  $X$  and  $Y$  is given by

$$f(x,y) = \begin{cases} \frac{1}{4}(2x+y) & \text{for } 0 < x < 1, 0 < y < 2 \\ 0 & \text{elsewhere} \end{cases}$$

find

- the marginal density of X;
- the conditional density of Y given  $X = \frac{1}{4}$ .

(6 marks)

6. (a) The table below shows the probability distribution function of a random variable X;

|      |   |      |   |     |      |
|------|---|------|---|-----|------|
| X    | 1 | 2    | 3 | 4   | 5    |
| P(x) | K | 1/12 | K | 1/2 | 1/12 |

Find;

- k
- $P(X \leq 2)$
- $P(3 \leq X < 5)$

(6 marks)

- (b) Let X be a random variable with probability function

$$f(x) = \begin{cases} \frac{2x}{12}, & x = 1, 2, 3 \\ 0 & \text{elsewhere} \end{cases}$$

Calculate

- $E(X)$
- $\text{Var}(X)$

(6 marks)