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NATIONAL OPEN UNIVERSITY OF NIGERIA

JABI, ABUJA
FACULTY OF SCEINCES
DEPARTMENT OF PURE AND APPLIED SCIENCE
JANUARY/FEBRUARY 2018 EXAMINATION

COURSE CODE: **PHY 313**
COURSE TITLE: **MATHEMATICAL METHODS FOR PHYSICS II**
TIME ALLOWED: **(3 HRS)**

INSTRUCTION: **Answer ONE and any other four (4) questions**

QUESTION 1

a. Prove that the sufficient condition for a function $f(z) = u + iv$ to be analytic at all points in a region R are
$\frac{\partial u}{\partial \lambda} = \frac{\partial v}{\partial y}$, $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$ & $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$ (8 Marks)
Are continuous functions of x and y in R.

b. Use the Cauchy-Rieman equation to show that $f(z) = z^3$ is analytic in the entire z-plane. (7 Marks)

c. Test the analyticity of the function $f(z) = \sin z$ and hence show that $\frac{d}{dz}\sin z = \cos z$.
(7 Marks)

QUESTION 2

a. Show that the real and imaginary parts of $f(z) = \log(z)$ satisfy the Cauchy-Rieman equations. (4 Marks)

b. Derive the polar form of the Cauchy-Rieman equations. (4 Marks)

c. Prove that for any analytic function $f(z) = u + iv$, both $u(x, y)$ and $v(x, y)$ are harmonic. (4 Marks)

QUESTION 3

a. If $w = \emptyset + i\psi$ represents the complex potential of an elective field and $\psi = x^2 - y^2 + \frac{x}{x^2+y^2}$ determine the function $\emptyset$. (4 Marks)

b. if $f(z) = u + iv$ is analytic and $u - v = e^x(\cos y - \sin y)$ find the $f(z)$ in terms of z. (4 Marks)

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c. Let $f(z) = u(r, \theta) + iv(r, \theta)$ be an analytic function. If $u = r^3 \sin 3\theta$, construct the corresponding analytic function $f(z)$ in terms of z. (4 Marks)

QUESTION 4

Determine the poles and the Residues at each pole of the following functions

a. $\frac{z^2}{(z-1)^2(z+2)}$

b. $\cot z$

c. $\frac{z^3}{(z-1)^4(z-2)(z-3)}$ at $|z| = 1$ only

QUESTION 5

a. Find the value of the integral $\int_0^{1+i}(x - y - ix^2)dz$

   i. 5ai Along the straight path from $z = 0$ to $z = 1 + i$ (4 Marks)

   ii. 5aii Along the real axis from $z = 0$ to $z = 1$ and then along a line parallel to the imaginary axis from $z = 0$ to $z = 1 + i$ (3 Marks)

b. Using Cauchy's integral theorem, find the value of

$$\int_C \frac{z+4}{z^2 + 2z + 5}$$

If C is the circle $|z + 1| = 1$ (5 Marks)

QUESTION 6

a. $\int_C \frac{1}{z} \cos z \, dz$ where C is the ellipse $9x^2 + 4y^2 = 1$ (4 Marks)

b. $\int_C \tan z \, dz$ where C is $|z| = 2$ (4 Marks)

c. $\int_C \frac{e^z}{z^2+1}$ where C is $|z| = 2$ (4 Marks)