NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, NnamdiAzikiwe Expressway, Plot 91, Cadastral Zone, Jabi, Abuja
FACULTY OF SCIENCES

JANUARY/FEBRUARY 2018 EXAMINATION

COURSE CODE: PHY309
COURSE TITLE: QUANTUM MECHANICS
COURSE UNIT: 3 Units
TIME: 3 hours
INSTRUCTION: Answer question one (1) and any FOUR(4) questions

Necessary Constants: $\hbar = 1.054\times10^{-34}\,Js$, $h = 6.63\times10^{-34}\,Js$, $m_e = 9.11\times10^{-31}kg$, $c = 3\times10^{8}ms^{-1}$ $h = 6.63x10^{-34}\,J.s$ $1eV = 1.6x10^{-19}\,J$

1. a). Show that the set $\left\{\begin{pmatrix}1\\0\\1\end{pmatrix},\begin{pmatrix}1\\1\\0\end{pmatrix},\begin{pmatrix}1\\2\\1\end{pmatrix}\right\}$ is linearly independent **(5 marks)**

b). Normalise each vector in the set $\left\{\begin{pmatrix}1\\2\\3\end{pmatrix},\begin{pmatrix}-2\\0\\4\end{pmatrix},\begin{pmatrix}1\\2\\1\end{pmatrix}\right\}$ **(10 marks)**

c). Check whether the following vectors are linearly independent $2i+3j-k, -i+j+3k \; and \; -3i+2j+k$.**(7 marks)**

2. a). If there exist a linearly independent set $[\phi_i]_{i=1}^{n}$, state the condition for ;

i. orthogonality ii. Orthonormality**(2 marks)**

b). Show that $\sin mx$ and $\sin nx$ are orthogonal, when $m \neq n$, for range $-\pi \leq x \leq \pi$ **(5 marks)**

c). Find the normalise function of the following

i. $\phi_1 = x$ ii. $\phi_2 = x^2 - \frac{1}{3}$ **(5 marks)**

3. a). Given the matrix $\begin{bmatrix} 3 & -2 \\ 2 & 2 \end{bmatrix}$, find the corresponding eigenvectors and the eigenvalues.**(5 marks)**

b). Find the eigenvalues and the corresponding eigenfunctions of the matrix $A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$. Hence determine the normalised wavefunction for each. **(5 marks)**

c). Given that kinetic energy operator for point energy $\hat{T} = \frac{-i\hbar d}{2mdx^2}$ and operator for momentum, $\hat{p} = -i\hbar \frac{d}{dx}$

Calculate:

i. $[\hat{T}, \hat{p}]$
ii. $[\hat{x}, \hat{p}]$

Give a brief comment/explanation in the result obtained in (i) and (ii).**(2 marks)**

4. a). i. What is photoelectric effect and give necessary equation **(2 marks)**

ii. With necessary equation explain Compton effect**(2 marks)**

b). Find the change in wavelength if a photon is scattered at an angle of 25^0 after its collision with an electron initially at rest.

(2 mark)

c). State 2 postulates of Bohr Theory of the Hydrogen atom.

(1 mark)

d). State Heisenberg's Uncertainty Principle.**(1 mark)**

e). i. Find the maximum kinetic energy with which an electron is emitted from ametal of work function $3.2\times10^{-39}\,J$ when a radiation of

energy $E = 3.313\times10^{-39}\,J$ falls on it, given that the work function is $3.2\times10^{-39}\,J$. **(2 marks)**

ii. What is the wavelength of the wave associated with an electron moving at $10^{6}\,m/s$. **(2marks)**

5.a). State the time-dependent Schroedinger equation for a free particle $(V=0)$ and hence by solving the time-dependent Schroedinger equation, find the condition imposed on the angular frequency and the wavenumber. **(6 marks)**

b). Which of the following functions would you recommend as a possibleeigenfunction in quantum mechanics?

i. $\Psi(x)=e^{-x^2}$ ii. $\Psi(x)=2x$ iii. $\Psi(x)=xe^{-2x^2}$ **(6 marks)**

6. a). State the correspondence principle **(3 marks)**

b). $\Psi(x)=A(ax-x^2)\,for|x|\le a.$ Normalise the wavefunction and find

i. $\langle x\rangle$ ii. $<x^2>$ andiii. $\Delta x.$ **(3 marks)**

c). A particle in a one-dimensional box $0\le x\le a$ is in state

$$\Psi(x)=\frac{1}{\sqrt{5a}}\sin\frac{\pi x}{a}+\frac{A}{\sqrt{a}}\sin\frac{x\pi x}{a}+\frac{3}{\sqrt{6a}}\sin\frac{3\pi x}{a}$$

i. Find A so that $\Psi(x)$is normalized. **(2 marks)**

ii. What are the possible results of measurements of the energy, and what arethe respective probabilities of obtaining each result? **(2 marks)**

iii. The energy is measured and found to be $\frac{9\pi^2\hbar^2}{2ma^2}$. What is the state of thesystem immediately after measurement? **(2 marks)**