NATIONAL OPEN UNIVERSITY OF NIGERIA
Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja

FACULTY OF SCIENCES
January\February Examination 2018

Course Code: MTH423
Course Title: Integral Equations
Credit Unit: 3
Time Allowed: 3 Hours
Total: 70 Marks
Instruction: Answer Question one and any other 4 Questions

1.(a) Briefly explain an integral equation **(4 marks)**

(b) Classify the following equations

(i) $u(x) = x - \frac{1}{6}x^3 - \int_0^x (x-t)u(t)dt = 0$ **(2 marks)**

(ii) $u(x) = \frac{1}{2} + x - \int_0^1 (x-t)\ u^2(t)dt = 0$ **(2 marks)**

(iii) $u'(x) = 1 - \frac{1}{3}x^3 + x - \int_0^x t\, u(t)dt = 0$ **(2 marks)**

(iv) $u(x) = \int_0^1 (x-t)^2\ u(t)dt = 0$ **(2 marks)**

(c) (i) Show that $u(x) = e^x$ is a solution of the equation

$u(x) = 1 + \int_0^x u(t)dt$ **(4 marks)**

(ii) Show that $u(x) = x$ is a solution of the integral equation

$u(x) = \frac{5}{6}x - \frac{1}{9} + \frac{1}{3}\int_0^1 (x+t)u(t)dt$ **(6 marks)**

2. (a) Convert the Volterra integral equation to an initial problem

$u(x) = x + \int_0^x (t-x)u(t)dt$ **(5 marks)**

(b) Solve the Fredholm integral equation

$u(x) = \frac{9}{10}x^2 + \int_0^1 \frac{1}{2}x^2t^2u(t)dt$ **(7 marks)**

3. (a) Using the transformation formula:

$$\int_0^x \int_0^{x_1} \int_0^{x_2} \cdots \int_0^{x_{n-1}} f(x_n)dx_n \cdots dx_1 = \frac{1}{(n-1)!}\int_0^x (x-t)^{n-1} f(t)dt :$$

Convert the quadruple integral to a single integral $\int_0^x \int_0^x \int_0^x \int_0^x u(t)dtdtdtdt$ **(2 marks)**

(b) Convert the following initial value problem to an equivalent Volteral integral equation

$$y''' - 3y'' - 6y' + 5y = 0 \qquad y(0) = y'(0) = y''(0) = 1$$ **(10 marks)**

4. Solve the Volterra integral equation $u(x) = x^2 + \frac{1}{12}x^4 + \int_0^x (t - x)u(t)dt$ by converting it to equivalent initial value problem **(12 marks)**

5. Convert the boundary value problem to an equivalent Fedholm integral equation

$$y''(x) + y(x) = x, \qquad 0 < x < \pi$$ **(12 marks)**

subject to boundary conditions $y(0) = 1, y(\pi) = \pi - 1$

6. (a) Solve the integral equation $3\sin x + 2\cos x = \int_{-\infty}^{\infty} \sin(x + y)Q(y)dy, -\pi \le x \le \pi$ (**8 marks)**

(b) Solve the integral equation $\int_0^x Q(x - y)[Q(y) - 2\sin ay]dy = x\cos ax$ **(4 marks)**