NATIONAL OPEN UNIVERSITY OF NIGERIA

Plot 91, Cadastral Zone, Nnamdi Azikiwe Expressway, Jabi, Abuja.

FACULTY OF SCIENCES

April/May Examination 2019

Course Code: **MTH412**
Course Title: **Functional Analysis II**
Credit Unit: **3**
Time allowed: **3 HOURS**
Total: **70 Marks**
Instruction: **ATTEMPT NUMBER ONE (1) AND ANY OTHER FOUR (4) QUESTIONS**

1. (a) Define Normed linear space. (6marks)
   (b) Show that the real line $\mathbb{R}$ becomes a norm linear space if you set $k \times k = |x|$ for every number $x \in \mathbb{R}$. (8marks)
   (c) Let $X = \mathbb{R}^2$ for each vector $\bar{X} = (x_1, x_2) \in X$ define $k - k_2 : X \to \mathbb{R}$ by $k\bar{X}k_2 = \sum_{k=1}^{2}(X_k^2)^{1/2}$. Then show that $k - k_2$ is a norm space. (8marks)

2. (a) Define a linear map or operators. (5marks)
   (b) Let $X = l_2$, for each $\bar{X} = (x_1, x_2, \ldots x_k, \ldots)$ in l_2 defined by $T\bar{X} = \left(0, x_1, \frac{x_2}{2}, \frac{x_3}{2}, \ldots\right)$. Then show that T is a linear map on l_2. (7marks)

3. (a) Consider the basis $\{v_1 = (2,1), v_2 = (3,1)\}$ on $\mathbb{R}^2$. Find the dual basis $\{f_1, f_2\}$ of $\mathbb{R}^*$. 5marks
   (b) Let $\{u_1, u_2, \ldots u_n\}$ be a basis of finite dimensional space X and let $\{\Phi_1, \Phi_2, \ldots \Phi_n\}$ be the dual basis in X^*. Then show that
   (i) For every vector $x \in X, x = \sum_{i=1}^{n} \Phi_k(x)u_k$
   (ii) For any linear functional $\sigma \in X^*, \sigma = \sum_{i=1}^{n} \sigma(u_k)\Phi_k$. (7marks)

4. (a) Define Equivalent norms. (5marks)
   (b) Prove that all norms defined on finite dimensional space are equivalent. (7marks)

5. (a) Define the following (i) Uniformly convergent (ii) Uniformly continuity (iii) Graph of a linear operator. (5marks)
   (b) Let T be an operator on Hilbert space H. Then prove that the following are equivalent
   (i) $T^*T = I$ (ii) $hTx, Ty_i = hx, y_i$ (iii) $kTxk = kxh$ for all $x \in H$. (7marks)

6. (a) When is a normed linear space said to be complete? (5marks)
   (b) Show that the space $C[a, b]$ of continuous real valued function on the interval $[a, b]$ is complete.
   (7marks)