NATIONAL OPEN UNIVERSITY OF NIGERIA
Plot 91, Cadastral Zone, Nnamdi Azikiwe Expressway, Jabi, Abuja.

FACULTY OF SCIENCES
January\February Examination 2018

Course Code:	**MTH411**
Course Title:	**Measure Theory & Integration**
Credit Unit:	**3**
Time Allowed:	**3 HOURS**
Instruction:	**ATTEMPT NUMBER ONE (1) AND ANY OTHER FOUR (4)**

QUESTIONS

1. (a) Let A⊂ [0, 1] be a non-measurable set. Let B ={ (x,0) ∈ R^2, x∈ A}.
(i) Is B a Lebesgue measurable subset of R^2? **1mark**
(ii) If the closure of E has Lebesgue measure zero, then E has Lebesgue measurezero. **2marks**

(b). If f: [0,1]→ R is continuous a.e, then f is measurable.
(i) Is the above statement true or false? **1mark**
(ii) Justify your answer in (i) above . **2marks**

(c). (i) Does there exist a non-measurable function such that $\sqrt{f}$is measurable? **1mark**
(ii) Justify your answer. **2marks**

(d). There is a subset A of R which is not measurable, but such that B={x∈A: x is irrational} is measurable.
(i) Is the above statement true or false? **1mark**
(ii) Justify your answer. **2marks**

(e). (i) Does there exist a non-measurable subset of R whose complement in R has outer measure zero? **1mark**
(ii) Support your answer. **2marks**

(f). Let f_n be a sequence of continuous functions Lebesgueintegrable on [0, ∞) which converges uniformly to a function fLebesgueintegrable on [0, ∞).
(i) Is it true that $\lim_{n\to\infty} \int_0^\infty |f(x) - f_n(x)|$dx = 0 **1mark**

(ii) Justify your answer **2½marks**

(g) .Let { $\int_k$ }be $\int_k \to \int$ a sequence of non-negative measurable function on **R** such that$\int_k \to \int$a.e in **R**. Then $\lim_{K\to\infty} \int R \int K$ exists and $\int R \int dm \leq \lim_{K\to\infty} \int R \int K dm$

(i) Is the above statement true or false? **1mark**
(ii) Justify your answer. **2½marks**

2(a) Prove that if two subsets A and B of the real line are measurable, then so is $A \bigcap B$ (6marks)

(b) Suppose f = g almost everywhere. Show that
$\int_{\chi} f d\mu = \int_{\chi} g d\mu$ (6marks)

(TOTAL = 12marks)

3 (a) Explain vividly what is meant by the Lebesgue Outer Measure μ*(E) of a subset E of the real line **R** 6marks

(b) Prove that for any two subsets A and B of **R**, if A $\subset$ B, then μ*(A) ≤ μ* (B) (6marks)

4(a) Explain carefully what is meant by the LebesqueOuter Measure μ*(E) of a subset E of the real line **R** **4marks**

(b) Prove that for any two subsets A and B of **R**, if A $\subset$ B, then μ*(A) ≤ μ* (B) **4marks**

(c) Let $E_1, E_2, E_3 \ldots, E_n$ be disjoint measurable subset of E with $\mu(E) < \infty$, then Every linear combination S = $\sum_{k=1}^{m} C_k \chi_{EK}$ with real coefficient $a_1, a_2, a_3, \ldots a_m$ is measurable simple function and $I_E(s) = \sum_{k=1}^{m} a_k \mu(E_k)$. **4marks**

5(a) Find the length of the set $\bigcup_{k=1}^{\infty}\left\{x : \frac{1}{k+1} \leq x \leq \frac{1}{k}\right\}$ **4marks**

(b) Prove that if E is any countable set of real numbers, the $\mu^*(E) = 0$ **4marks**

(c) Prove that for every countable family $\{E_n\}_{n=1}^{\infty}$ of subsets of **R,** we have $\mu^*\left(\bigcup_{n=1}^{\infty} E_n\right) \leq \sum_{n=1}^{\infty} \mu^*(E_n)_{()}$ **4marks**

6(a) Show that a subset E of **R'** is Lebesque measurable if and only if every subset X of **R'**we have
$\mu^*(X) = \mu^*(X \cap E) + \mu^*(X \setminus E)$ **6marks**

(b) Prove that if a subset E of **R'** is measurable, then so is its complement **6marks**