NATIONAL OPEN UNIVERSITY OF NIGERIA

Plot 91, Cadastral Zone, Nnamdi Azikiwe Expressway, Jabi, Abuja.

FACULTY OF SCIENCES

April/May Examination 2019

Course Code:	**MTH402**
Course Title:	**General Topology II**
Credit Unit:	**3**
Time allowed:	**3 HOURS**
Total:	**70 Marks**
Instruction:	**ATTEMPT NUMBER ONE (1) AND ANY OTHER FOUR (4) QUESTIONS**

1. (a) Define topology on a set (3marks)
(b) Prove that the intersection $\tau = \bigcap_{\alpha} \tau_{\alpha}$ of topologies $\{\tau_{\alpha}\}_{\alpha\in\Delta}$ on X is itself a topology in X (where Δ is some index set). (7marks)

(c) Define a basis for topology (5marks)

(d) Let X be a set and B be a basis for a topology τ on X. Then prove that τ equal to the collection of all unions of elements of B. (7marks)

2. (a) Define closure and interior of a set. (5marks)
(b) Prove that if A is a subset of a topological space X, then $x \in \bar{A}$ if and only if every neighbourhood of x intersects A i.e., $x \in \bar{A}, V \in \aleph(x), V \cap A = \emptyset$. (7marks)

3. (a) Define Hausdorff space and give two examples. (5marks)
(b) Let X be a Hausdorff space, then prove that a sequence of points of X converges to at most one point of X (i.e., if a sequence $\{x_n\}$ in X, is a Hausdorfff space, converges, the limit is unique). (7marks)

4. (a) Define the following: (i) Dense set (ii) Baire space. (iii) Countable sets. (5marks)
(b) Let (X, τ) be a topological space and let A be a subset of X. Then show that A is dense in X if and only if every nonempty open set subset of $X, A \cap U = \emptyset$. (7marks)

5. (a) Differentiate between connected sets and separated spaces (5marks)
(b) Let $\Re$ be endowed with the standard topology. Then show that for all $x \in \Re$
$\omega = \{(x - \varepsilon, x + \varepsilon), \varepsilon > 0\}$ is a neighbourhood basis of x. (7marks)

6. (a) Define a dense set. (5marks)
(b) Show that $\mathbb{Q}$ the set of rational numbers is dense subset of $\mathbb{R}$ because $\overline{\mathbb{Q}} = \mathbb{R}$. (7marks)