NATIONAL OPEN UNIVERSITY OF NIGERIAN
Plot 91, Cadastral Zone, NnamdiAzikiwe Express Way, Jabi-Abuja

FACULTY OF SCIENCES
January\February Examination 2018

Course Code:MTH401
Course Title: General Topology I
Credit Unit: 3
Time Allowed: 3Hours
Total Marks: 70%

INSTRUCTION: ANSWER QUESTION ONE(1) AND ANY FOUR (4) QUESTIONS (TOTAL = 5 QUESTIONS IN ALL)

1(a) Let p be a prime number, and d : Z x Z → [0;+∞) be a function defined by
$d_p(x; y) = p^{-\max\{m\varepsilon N:\ pm/x - y\}}$. **(4marks)**

1(b) Let (X, τ) be a topological and (Y, d) a metric space. If f, g : X→ Y are continuous, show that the set $\{x_1 \in X; f(x) = g(x)\}$ is closed. **(4½marks)**

1(c) Let (X; d) be a metric space and F $\dot{\subseteq}$ X be a finite subset. Prove that F is closed in X. **(2½marks)**

1(d) Let R be endowed with standard topology. Show that for all x ε R, w = { (x – ε, x + ε), ε > 0} is a neighbourhood basis of x. **(4½marks)**

1(e) Let $\tau_1$and τ_2 be two topologies on the same space X.

(i) Show that $\tau_1 \subseteq \tau_2$ if and only if given x∈τ_1 there exists v ε τ_2 such that x ε V ⊆U **(2½marks)**

(ii) Show also that $\tau_1 = \tau_2$ if and only if given x ∈ U∈τ there exists v ∈$\tau_2$and x∈V ⊆U and given x ε U ∈τ_2 , we can find U ∈ τ_2 such that x∈V =U. **(4marks)**

(TOTAL=22marks)

2(a) Consider the interval [0, 1] with the Euclidean metric and A = [0, 1] ∩ Q with the inherited metric. Exhibit,and prove, a continuous map f: A →R (where R has the standard metric) such that the set { x_1∈ X ; f(x) = g(x)} is closed **(6marks)**

2(b) Let (X; d) be a metric space and Y $\subseteq$ X be a subspace. Then U $\subset$ Y is open (closed) in Y if and only if there exist V$\subseteq$ X open (closed) in X such that U = V ∩ Y .**(6marks)**

3(a) Consider (R, τ) with the standard (Euclidean) topology, then show that a set E is compact if and only if it is closed, and bounded (that is, there exists an M such that

$|x| \leq$ M for all x ε E) **(6marks)**

3(b) Let (X; d) be a metric spaceand;Φ ≠ Y $\dot{\subseteq}$ X. Prove that the function X → R defined by x → d(x; Y) is a continuous function. **(6marks)**

4(a) If (X, τ) and (Y, σ) are Hausdorff topological spaces, then show that X x Y with the product topology is also Hausdorff. **(4marks)**

4(b) Show that the product of two Hausdorff spaces is Hausdorff **(4marks)**

4(c) Prove that a discrete metric space is compact if and only if its underlying set is finite **(4marks)**

5(a) Let $\{ X_\alpha \}_{\alpha \varepsilon J}$ be an indexed family topological spaces. Assume that the topology on each

X_α is given by a basis B_α. Show that the family $\{ \prod_{\alpha \varepsilon J} B_\alpha : B_\alpha \varepsilon B_\alpha$ and $B_\alpha = X_\alpha$ for all but finitely many $\alpha \varepsilon J\}$ is a basis for the product topology $\prod_{\alpha \varepsilon J} X_\alpha$ (7marks)

5(b) Let X be a set and T = {U ⊆X : U = Φ ; or X\U is countable}. Prove that T is a topology on X **(5marks)**

6(a) Let X be a set and B be a basis for a topology on X. Prove that the topology generated by B is indeed a topology.**(3marks)**

6(b) Let X be the set of all sequences x: N→ R converging to zero. Show that:**(3marks)**

(i) The function d : X xX→ [0, +∞] defined by $d(x_n, y_n) = \sup_{n \varepsilon \mathbb{N}} (x_n, y_n)$is a metric on X **(3marks)**

(ii) The metric space (X, d) is separable. **(3marks)**