



NATIONAL OPEN UNIVERSITY OF NIGERIA

Plot 91, Cadastral Zone, Nnamdi Azikiwe Expressway, Jabi, Abuja.

FACULTY OF SCIENCES

April/May Examination 2019

Course Code: MTH401
Course Title: General Topology 1
Credit Unit: 3
Time allowed: 3 HOURS
Total: 70 Marks
Instruction: ATTEMPT NUMBER ONE (1) AND ANY OTHER FOUR (4) QUESTIONS

1. (a) Define a metric space (4marks)
 (b) Let \mathbb{R} denote the set of real numbers and let $d: \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ be defined by $d(x, y) = |x - y|$ for all $x, y \in \mathbb{R}$. Show that d is a metric on \mathbb{R} . (6marks)
 (c) State Triangle and Hölder's inequalities (4marks)
 (d) State and prove Minkowski's inequality. (8marks)

2. (a) Define the following: (i) Open ball (ii) Closed ball (iii) Spheres. (5marks)
 (b) Let $E = \mathbb{R}$ be endowed with the Euclidean metric.

$$d_2(X, Y) = \sum_{i=1}^k \{(X_k - Y_k)^2\}^{1/2}$$
 for all $x = (x_1, x_2), y = (y_1, y_2) \in \mathbb{R}^2$.
 Describe the following sets (i) $B((0,0), 1)$ (ii) $\bar{B}((0,0), 1)$ (iii) $S((0,0), 1)$ where $(0,0) \in \mathbb{R}^2$ (iv) $B_r(x_0)$ for arbitrary $x_0 \in \mathbb{R}^2$. (7marks)

3. (a) Define the closure of a set. (5marks)
 (b) Every singleton subset of any metric space is closed. Hence, every finite set is closed. (7marks)

4. (a) Let $\{x_n\}_{n=1}^{\infty}$ be a sequence of points in a metric space (E, d) . When is a point $x \in E$ said to be a limit point of the sequence $\{x_n\}$? (5marks)
 (b) Show that $\{x_n\}$ converges to x in E , if and only if $\{d(x_n, x)\}$ converges to 0 in \mathbb{R} . (7marks)

5. (a) When is a sequence said to be a Cauchy in a metric space? (5marks)
 (b) Prove that every convergent sequence in a metric space is Cauchy. (7marks)

6. (a) Define a connected space (5marks)
 (b) Prove that the image of a connected space under a continuous map is connected. (7marks)