NATIONAL OPEN UNIVERSITY OF NIGERIA
Plot 91, Cadastral Zone, NnamdiAzikwe Express Way, Jabi-Abuja

FACULTY OF SCIENCES
January\February Examination 2018

Course Code: MTH382
Course Title: Mathematical Methods IV
Credit Unit: 3
Time Allowed: 3 Hours
Total: 70 Marks
Iinstruction: Answer Question one and Any other4 Questions

1. (a) The gamma function is defined by the improper integral as $\Gamma(x) = \int_0^\infty t^{x-1} e^{-t}\, dt$,

   (i) if $x = \frac{1}{2}$, prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$(9 marks)

   (ii)Evaluate$\Gamma\left(\frac{7}{2}\right)$(3 marks)

   (b) The beta function $B(m,n)$is defined by$B(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\, dx$,

   (i) if $x = \sin^2\theta$, prove that $B(m,n) = 2\int_0^{\frac{\pi}{2}} \sin^{2m-1}\theta \cos^{2m-1}\theta\, d\theta$ (6 marks)

   (ii) evaluate$2\int_0^{\frac{\pi}{2}} \sin^7\theta \cos^3\theta\, d\theta$(4 marks)

2. (a) Use the generating function $f(t,x) = \frac{1}{\sqrt{1-2xt+t^2}} = \sum_{n=0}^\infty P_n(x)\, t^n$ to obtain the recurrence

   relation$(2n+1)xP_n(x) = (n+1)P_{n+1}(x) + nP_{n-1}(x)$(6 marks)

   (b) Use the recurrence $(2n+1)xP_n(x) = (n+1)P_{n+1}(x) + nP_{n-1}(x)$ to obtain the first four

   Legendre polynomials   (6 marks)

3. (a) Solve the Hermite differential equation$\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$(8 marks)

   (Hint: use Frobenius method)

   (b) Evaluate the followings (i) $\frac{\Gamma(5)}{2\Gamma(3)}$(2 marks)

                                        (ii) $B(6,5)$(2 marks)

4. (a) Use the Rodrigue's formula $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n}\left(x^2 - 1\right)^2$ to drive Legendre polynomial $P_4(x)$

   (5 marks)

   (b)Using the Rodrigue' formula, show that (i) $f(x) = x^2$ (ii) $f(x) = 1 + x + x^3$ can be written as a

      series of Legendre polynomials   (7 marks)

5. (a) By the recurrence relation $T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x)$, derive $T_6(x)$ **(7 marks)**

   (b) Express the polynomial $16x^4 + 12x^3 + 6x^2 + 4x - 1$ in terms of $T_n(x)$ **(5 marks)**

6. (a) Find $\frac{\partial z}{\partial x}$ for the following functions

   (i) $3z^4 + x^7 = 5x$ **(2 marks)**

   (ii) $x^3z^2 - 5xy^3z = x^2 + y^2$ **(3 marks)**

   (b) If $u(x,y) = e^x \cos y$ and $u(x,y) = e^{-x} \sin y$, find the derivative of

   (i) $\quad \frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$ **(2 marks)**

   (ii) $\quad \frac{\partial v}{\partial x}$ and $\frac{\partial v}{\partial y}$ **(2 marks)**

   Hence, evaluate the Jacobian of $u(x, y)$ and $v(x, y)$ **(3 marks)**