NATIONAL OPEN UNIVERSITY OF NIGERIA
Plot 91, Cadastral Zone, NnamdiAzikiwe Expressway, Jabi, Abuja.

FACULTY OF SCIENCES
January\February Examination 2018

Course Code: **MTH381**
Course Title: **Mathematical Methods III**
Credit Unit: **3**
Time Allowed: **3 HOURS**
Instruction: **ATTEMPT NUMBER ONE (1) AND ANY OTHER FOUR (4) QUESTIONS**

1. (a) *If* $f(x,y)=\dfrac{4x+2y}{2-2xy}$; *find f(1,-3)* **[5 Marks]**

(b) If $u=(x,y,z), v=x+y+z; w=x+y+z$

Find the Jacobian $J=\dfrac{\delta(u,v,w)}{\delta(x,y,z)}$ **[5 Marks]**

(c) Evaluate the Laplace transform of $\cos\omega t$ **[6 Marks]**

Evaluate the double integral $I=\int_0^4\int_y^{2y}(2x+3y)dxdy$ **[6 Marks]**

2. **(a)** Evaluate $\int_{-\pi i}^{\pi i}\cos zdz$ **[6 Marks]**

. (b) Determine the Fourier Series to represent the function:

$$f(x)=\begin{cases}0 & -\pi \prec x \prec -\pi/2 \\ 3 & -\pi/2 \prec x \prec \pi/2 \\ 0 & \pi/2 \prec x \prec \pi\end{cases};$$ **[6 Marks]**

$$f(x+2\pi)=f(x)$$

3. (a) *Given that* $u(x,y) = e^{-x}\cos y$, show that $u(x,y)$ is an harmonic function and then find the function $v(x,y)$ that is conjugate to $u(x,y)$.

(Hint: find $v(x,y)$ that ensures that $f(x) = u(x,y) + jv(x,y)$ is analytic). **[6 Marks]**

(b) State and prove Liouville's theorem. **[6 Marks]**

4. (a) Evaluate $\iiint_R \left[\frac{xy - y^2}{z}\right] dxdydz$, where R is the rectangular box

$1 \le x \le 3, 0 \le y \le 2, and\, 1 \le z \le 2,$ in space. **[4 Marks]**

(b) If $z_1 = 3 + 5i$ and $z_2 = 2 + 3i$

Find (i) $z_1 + z_2$ (ii) $|z_1 + z_2|$ (iii) z_2/z_1 (iv) $\bar{z}_1$ (v) $z_1\, z_2$ **[8 Marks]**

5. (a) (i) State the Green's theorem **[2 Marks]**

(ii) Let $M = xy^2$ and $N = x^4 + x^2 y$ and let C be the circle $x^2 + y^2 = 1$

oriented anticlockwise;

Find $\int_C M(x,y)dx + N(x,y)dy$ by applying the Green's theorem. **[6 Marks]**

(b) Evaluate $f(z) = \frac{1}{z^3 - z^4}$ around the circle C in the clockwise sense, where $|z| = 1/2$

[6 Marks]

6. (a) Let $F(x,y,z) = 6xi + 6yj + 4zk$

Evaluate $\iiint_D divF(x,y,z)\,dv$ using the Divergence theorem, where D is the ball:

$x^2 + y^2 + z^2 \le 1$ **[6 Marks]**

(b)Write the complex number $z = x + iy$ in the polar form; hence find *mod z and arg z.*

[6 Marks]